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IONIZATION AND EXCITATION OF IONS BY ELECTRON IMPACT
-REVIEW OF EMPIRICAL FORMULAE-

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Ionization and Excitation of Ions by Electron Impact

-Review of empirical formulae-

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We review here the empirical formulae for ionization and excitation by electron impact, especially for highly-ionized ions. The reliability and applicability of various empirical formulae are discussed.

A. Excitation

The excitation cross sections for ions have not been investigated so much on either theoretical or experimental side, but it is often necessary to know cross sections for plasma diagnostics and for astrophysics. So it is convenient to find a empirical formula in order to calculate cross sections easily.

In order to calculate line intensities from a hot plasma, we have to know the cross section for excitation. When the excitation from levels other than ground state and the cascade effect are neglected, the intensity P_{ji} of transition from level j to i is expressed as

$$P_{ji} = N_g N_e \alpha_{gj} D_{ji} h\nu, \tag{1}$$

where α_{gj} is the rate coefficient of collisional excitation from ground state to excited level j, $D_{ji} = A_{ji} / \sum_{\ell<j} A_{j\ell}$ the

branching ratio, N_g the density of the ion, N_e the electron density and A_{ji} is the spontaneous radiative transition probability. α_{gj} can be obtained by integrating the cross section σ over a Maxwellian distribution,

$$\alpha_{gj} = \int f(v)\sigma v dv. \tag{2}$$

The collision strength are defined by $\Omega = k_i^2\omega_i\sigma$, where σ the cross section in unit of πa_o^2, k_i^2 the energy of the incident electron in redbergs associated with level i, ω_i is the statistical weight of level i. As the collision strength Ω is a slowly varying function of energy, taking the averaged collision strength $\overline{\Omega}$

$$\alpha_{gj} = \frac{8.63\times10^{-6}}{\omega_g}\,\overline{\Omega}\,T^{-1/2}(^\circ K)\exp(-E_{gi}/kT)\,cm^3 sec^{-1}, \tag{3}$$

where ω_g is the statistical weight for the ground state, T is the electron temperature, and E_{gj} is the excitation energy.

1. Approximation Method

We survey here the approximation methods in a simple way. More detailed reviews are found in Bely and Van Regemorter (1970).

(a) Quantum theory

The total wavefunction Ψ is expanded in terms of the multiple of unperturbed atomic eigenfunction $\phi_n(\gamma_1\cdots\gamma_\ell)$ and of the wavefunction $\chi_n(\gamma)$ of incident electron, like

$$\Psi(\gamma_1\cdots\cdot\gamma_\ell,\ \gamma) = A\sum_n\phi_n(\gamma_1\cdots\cdot\gamma_\ell)\chi_n(\gamma). \tag{4}$$

It is necessary to make a partial-wave analysis in which the differential equations are solved for each value of the total angular momentum and of the total spin in order to make accurate calculations. This type of analysis requires a long time of computation.

(a-1) The close-coupling approximation

This is the approximation assumed only a finite number of state in the expansion (4). Then one has to solve a finite set of coupled integrodifferential equations using numerical techniques.

(a-2) Born approximation

When the incident energy is large compared with the interaction energy, the wavefunction may be approximated by a plane wave and the cross section is proportional to

$$|\langle\Psi^{+}_{n'}|v|\Psi_{n}\rangle|^{2}. \tag{5}$$

$\Psi_n = \phi_n F_n$ and $\Psi_{n'} = \phi_{n'} F_{n'}$, where F_n and $F_{n'}$ are plane waves. Born approximation is valid for high incident energies. For collision with positive ions, F_n and $F_{n'}$ should be taken as Coulomb waves. This is called the Coulomb-Born approximation. Born approximation which takes into account the distortion of the wavefunction by the static atomic field is called the distorted-wave Born approximation. There are several other improved Born approximation (for example see Bely and Van Regemorter (1970)).

Vainshtain and Sobelman (1968) made calculations for the cross sections of neutral atoms and listed their results in Tables. They introduced a approximate analytical expressions for the cross section,

$$\sigma = \pi a_o^2 \left(\frac{Ry}{E_{ij}}\right)^2 \left(\frac{E_1}{E_o}\right)^{3/2} \frac{QC}{(2\ell_o+1)(w+\phi)} \tag{6}$$

where E_{ij}: threshold energh,

E_o : ionization potential for the initial state,

E_1 : ionization potential for the final state,

ℓ_o : the electron angular momentum of the initial state,

Q : depends on the transition (the orbital angular momentum of an electrons ℓ and the total orbital angular momentum L), the expression of Q for each transition is given in the text (Q ≈ the number of electrons),

E : incident energy,

w : $\frac{E - E_{ij}}{E_{ij}}$

The rate coefficient is

$$\alpha = 10^{-8} \left(\frac{Ry}{E_{ij}} \cdot \frac{E_1}{E_o}\right)^{3/2} e^{-y} \frac{Q}{2\ell_o+1} \frac{A\sqrt{y}(y+1)}{y+\chi}, \tag{7}$$

where $y = E_{ij}/\kappa T$. The values of c, ϕ, χ, Q and A are given in Table. This approximation is used by Beigman, Vainshtain and Vinogradov (1970). For the intercombination transition ($\Delta s \neq 0$), the approximation is not good because they used the Ochkur (1963)'s method which is not so reliable in the case of spin exchange transitions.

(a-3) Glauber approximation

This is the method for the scattering on a target which consists of many body, and the dynamics of a many body system is considered.

Good results are obtained for the atomic scattering (for example Narumi and Tsuji (1975)). For low energies, smaller values are obtained than by Born approximation.

(a-4) Bethe approximation

At high energies, the distant encounters are important and the colliding electron remains outside the atom most of the time. Bethe asymptotic formula for optically allowed transitions is expressed simply using the absorption oscillator strength

$$\sigma = 4\pi a_0^2 (I_H/E_{ij})^2 f_{ij} \; u^{-1} \; \ln(bu),$$

where $u = E/E_{ij}$ and I_H is the hydrogen ionization energy (13.6 eV). Seaton (1962a) introduced the effective Gaunt factor g(u) and gave the following formula

$$\sigma = \frac{2\pi}{\sqrt{3}} \cdot 4\pi a_0^2 \left(\frac{I_H}{E_{ij}}\right)^2 f_{ij} \; g(u) \; u^{-1} = 1.28\times10^{-15} \left(\frac{I_H}{E_{ij}}\right)^2 f_{ij} \; g(u) \; u^{-1}. \tag{8}$$

This is sometimes called the g empirical formula.

(b) Semiclassical theory

(b-1) Impact-parameter method

Seaton (1962b) introduced the semiclassical impact-parameter method for atom-electron collisions, and the results are much better than Born approximation at low energies. The results should be reliable to within 50 % for strong allowed transitions from the ground state or between n → n + 1 levels. This method requires a knowledge of the oscillator strength f. If the colliding electron remains outside the target, the dipole approximation is valid, and this gives the same expression as Bethe formula.

II. g empirical formula by Bethe approximation

This approximation is often used as it is simple and practical. There are several methods depending on the form of the gaunt factor g(u) in eq.(8), and typical examples which are currently used are discussed. We use the same notation as in the section before; $u = E/E_{ij}$ and $y = E_{ij}/kT$

(1) Post (1961)

For the case of high velocity impacts, he assumed

$$g(u) \simeq \ln u, \tag{9}$$

and the rate coefficient is

$$\alpha = \frac{4.4\times10^{-6}}{T_e^{3/2}(\mathrm{ev})} f_{ij} h(y) \quad \mathrm{cm}^3\mathrm{sec}^{-1}, \tag{10}$$

where

$$h(y) = \frac{\varepsilon E_1(y)}{y} \qquad E_1(y) = \int_y^\infty \frac{e^{-z}}{z}\,dz$$

$$y = E_{ij}/kT \qquad \varepsilon = 2.7183$$

and T_e is the electron temperature in eV.

(2) Van Regemorter (1962), Allen (1973)

This formula has been used often in astrophysics.

$$\sigma = 1.28 \times 10^{-15} \frac{1}{u}\left(\frac{I_H}{E_{ij}}\right)^2 f_{ij}\ \bar{g} \tag{11}$$

$\bar{g}$ is a function of $x = \sqrt{E/E_{ij}}$ and its values are given in Table. $\bar{g}$ can be taken to be a constant ~ 0.2 at low energies for positive ions. But from the recent results of Coulomb-Born I and II methods, $\bar{g}$ is generally larger than 0.2 for the transition of the same principal quantum numbers ($n = n'$) and smaller than 0.2 when $n \neq n'$. This expression of $\bar{g}$ has been improved by Mewe (1972) as will be discussed later.

The rate coefficient is

$$\alpha = 1.70\times10^{-3} f_{ij}\ T^{-1/2}(°K)\ E_{ij}^{-1}(ev)e^{-y}P(y) \tag{12}$$

The values of

$$P(y) = \int_0^\infty \bar{g}(x)\ e^{-yx^2} d(yx^2),$$

are given in Table.

(3) Drawin (1966)

$$g(u) = \begin{cases} 0.083\,u & \text{for } 1 \le u \le 3.85 \\ \dfrac{0.275\times(u-1)}{u}\,\ell_n(1.25\,u) & \text{for } u > 3.85 \end{cases} \tag{13}$$

and

$$\alpha = 8.69\times10^{-8}\ \beta_{ij}\, f_{ij}\ Z_{eff}^{-3}\ \frac{y_1^{3/2}}{y}\ \Psi_2(y) \tag{14}$$

where

$$\Psi_2(y) = \frac{0.302}{y}\int_y^{3.85y} z\,e^{-z}dz + \int_{3.85y}^{\infty}[(1-\tfrac{y}{z})e^{-z}\ell n(1.25-\tfrac{z}{y})]dz \tag{15}$$

$\beta_{ij} = 0.8 \sim 1.2$

$y_1 = 1.58 \times 10^5\ Z^2/T(°K)$

The values of $\Psi_2(y)$ are given in Table in the text and Z_{eff} is the effective charge $Z_{eff} = Z - N_e + 1$

(4) Mewe (1972)

He assumed

$$g(u) = A + Bu^{-1} + Cu^{-2} + D\ \ell n\ u. \tag{16}$$

D is taken to be $\sqrt{3}/2$ according to the Bethe limit in the case u >> 1.

$$\alpha = 1.70\times10^{-3}\ T^{-1/2}(°K)\ E_{ij}^{-1}\ f_{ij}\ P(y)\ e^{-y} \tag{17}$$

and

$$P(y) = y\, e^{y} \int_{1}^{\infty} g(u)\, e^{-yu} du$$
$$= A + (By - Cy^{2} + D)\, e^{y}\, E_{1}(y) + Cy,$$

where

$$E_{1}(y) = \int_{y}^{\infty} \frac{e^{-z}}{z}\, dz$$

It can be written within 3.5 % accuracy:

$$e^{y}\, E_{1}(y) \approx \ln\left(\frac{y+1}{y}\right) - \frac{0.4}{(y+1)^{2}}$$

Mewe got the values of coefficients A, B, C, D, for several transitions of H-, He-, Li- and Ne- like ions by fitting the theoretical and experimental results available. We listed them in Table I. For all other sequences,

(i) $A = 0.15$, $B = C = 0$, $D = 0.28$ (allowed, $\Delta n \neq 0$)
(ii) $A = 0.6$, $B = C = 0$, $D = 0.28$ (allowed, $n = 0$)
(iii) $A = 0.15$, $B = C = D = 0$ (forbidden monopole or quadrupole transition)
(iv) $A = B = D = 0$, $C = 0.1$ (spin exchange transition)

are to be taken. In the case of a forbidden transition, f_{ij} values are defined as that of the nearest allowed transition. For example, for the transitions 1s - ns and 1s - nd, the f_{ij}-value of 1s - np is taken.

From eq.(3) and (17), we get

$$\overline{\Omega} = 1.97\times10^{2}\, f_{ij}\, P(y)\, \omega_{i}/E_{ij}. \qquad (18)$$

We compare in Table II the value $\overline{\Omega}_{M}$ which is calculated from eq.(18) with the more accurate value which is obtained theoretically or experimentally. The average value is $<\overline{\Omega}_M/\Omega>$ $= 1.9$. We get good agreement for the allowed transitions near the ground state.

P(y) for the sequences other than H-, He-, Li and Ne-sequences in the case of (i), (ii), (iii) and (iv) are shown in Fig.1 as a function of $y = E_{ij}/kT$.

This empirical formula by Mewe is considered as the best of the formulae discussed above in (1), (2), (3) and (4).

III. Comparison

We compared the effective Gaunt factor $g(u)$ for the four empirical formulae discussed in §II and showed them in Fig.2 as a function of $u = E/E_{ij}$.

The comparison of the excitation rate coefficient is shown in Fig.3(a) for the transitions of NeVII (Be-seq.) $2s^2(^1P)$ - $2s2P(^1P)$ and of $2s^2(^1P)$ - $2s2p(^3P)$. The dotted line shows the result obtained by eq.(3) using the collision strength Ω, calculated theoretically by Osterbrock (1970). Black points are the experimental results by Johnston III and Kunze (1971), for which the error may be about a factor of 2. As seen in Fig.3(a), the formula of Drawin (1966) gives the largest values and that of Van Regemorter (1962) the smallest. As Mewe (1972) gave the values of coefficients A, B, C and D for the H, He, Li, and Ne-sequences (see §II (4)), we choose Be-sequence for comparison. Much better agreement would be expected for transitions of H, He, Li and Ne-sequences that for other sequences in the case of Mewe's formula. The comparison of the rate coefficient for the transitions of OVII $1s^2(^1S)$ - $1s2p(^1P)$ and of $1s^2(^1S)$ - $1s2p(^3P)$ is shown in Fig.3(b). There is not so much difference between the values in the temperature range of $10^6 \sim 10^7$ °K. The experimental results by Elton and Köppendörfer (1967) are also shown in Fig.3(b). All the calculated values are in good agreement with the experimental values within an experimental error smaller than a factor of 2.

B. Ionization

Ionization theory is much more difficult than excitation theory, since there are now two free electrons to consider, the ejected electron and the scattered one, after an ionizing collision. But the results obtained by various approximations are in better agreement with the experimental data than for excitation. After Van Regemorter (1970), 50 % accuracy is generally reached, and empirical formulae with an accuracy of a factor 2 can be given easily for ionization while it is difficult for excitation.

I. Approximation method

(a) Classical theory

Thomson (1912) introduced the following simple formula for the first time

$$\sigma = 4\zeta(I_H/I)^2 \ u^{-1} \ (1 - u^{-1}) \ \pi a_o^2, \qquad (19)$$

where ζ is the electron number of a shell with binding energy I, $u = E/I$ and I_H is the ionization energy of hydrogen.

If we define a reduced cross section by

$$\sigma^R(u) = (I/I_H)^2 \zeta^{-1} \sigma, \tag{20}$$

$\sigma^R(u)$ is then a function of u only:

$$\sigma^R(u) = 4u^{-1}(1 - u^{-1})\pi a_o^2 \tag{21}$$

It is important that a reduced cross section depends only on energy but not on other atomic parameters. For high energies $\sigma \sim \log E/E$ due to the quantum effect, and eq.(19) is not valid. Thomson's theory gives linear threshold low which agrees with the quantum mechanical results (Rudge & Seaton (1965)).

Gryzinski (1965) derived a fairly simple expression for the ionization cross section of atoms, improving the classical theory,

$$\sigma = \sigma_o \, q(u)\zeta/I^2 \tag{22}$$

where

$$\sigma_o = Z^2 \times 6.56 \times 10^{-14} \text{ eV}^2 \text{ cm}^2$$

$$g(u) = \frac{1}{u}\left(\frac{u-1}{u+1}\right)^{3/2}\left\{1 + \frac{2}{3}\left(1-\frac{1}{2u}\right)\ell n[2.7+(u-1)^{1/2}]\right\}$$

(b) Semiclassical theory

Burgess (1963, 1964) improved Thomson's theory by taking into account some quantum-mechanical properties, particularly exchange between the two electrons. Furthermore he combined this theory with the high energy results given by the impact parameter method. This is the same method that we already discussed in §A, I(b) for the case of excitation, and it requires the knowledge of the oscillator strength f(free-bound). The agreement with experimental data is good.

(c) Quantum theory

In the case of ionization, it is difficult to find the asymptotic form of the field, since both ejected and scattered electrons are in continuum state. There are several approximation methods used in the quantum theory, and they are reviewed in articles by Rudge & Seaton (1965) and Rudge (1968). We note here only the characteristics of some approximation methods.

- Born approximation (a), (b); Born (b) is extensively used at present. If the wavefunction describing the bound state of the target is good, we can expect 50 % of accuracy.
- Born Oppenheimer approximation gives better results for highly ionized ions.
- Born-exchange approximation; Good agreement between theory and experiment is obtained for H(1s) and for $He^+(1s)$.
- Born (b) and Born-exchange are considered as the best available approximations now, but recently Glauber approximation has also been used for the ionization problem and has given good results.
- For high energies, from the Bethe form

$$\sigma = A \log(E)/E + B/E. \tag{23}$$

Odmivar (1969) gives analytical expressions for A and B for hydrogenic systems in their ground states. Inokuti and Kim (1968) gave accurate values of A and B fro H^-.

II. Empirical formula

It is not as difficult to find an empirical formula in the energy range $E/I \lesssim 10$ as in the case of excitation, whereas the theory itself is more difficult. As we can suppose from eq.(21), the reduced cross section might not depend on the species of ion. In fact almost all of theoretical and experimental reduced cross sections σ^R agree with each other within a factor of 2. Most of the empirical formulae are based on this result. The scatter of the reduced cross sections increases at high energies.

Elwert gave first a simple empirical formula and many other formulae have been proposed later. We discuss here several typical formulae.

(1) Post (1961)

The cross section is expressed by

$$\sigma = \frac{\pi e^4}{I^2} b \frac{\varepsilon}{u} \ln u = \frac{0.4\times10^{-13}}{I^2(\mathrm{eV})} \frac{\ln u}{u} \ \mathrm{cm}^2, \qquad (24)$$

where $u = E/I$, and the ionization rate coefficient β is

$$\beta = \frac{4\pi e^4}{(2\pi mkT)^{1/2}kT} b\, h(x) = \frac{1.4\times10^{-4}}{T_e^{3/2}(\mathrm{eV})} b\, h(x) \ \mathrm{cm}^3\ \mathrm{sec}^{-1}, \qquad (25)$$

where $x = I/kT$. The definitions of $h(x)$ is the same as eq.(10), and $b \approx 0.2$.

(2) Drawin (1966)

The cross section for Z and for i-state is

$$\sigma = 2.34 \times 10^{-16}\left(\frac{I_1^H}{I}\right)^2 \xi_i \frac{u-1}{u^2}\, \ell n(1.25\gamma_i u)\ \mathrm{cm}^2. \tag{26}$$

The values ξ_i are listed on Table for the case of $i = 1$ (ionization from the ground state). For all other excited states ($i > 1$), $\xi_i \cong 1$. The value of γ_i is of the order of 1 and is written approximately

$$\gamma_i = 1 + \frac{Z_{eff} - 1}{Z_{eff} + 2}, \tag{27}$$

where $Z_{eff} = Z - N_e + 1$ is the effective charge seen by the electron in i-state.

The ionization rate coefficient is

$$\beta = 1.46\times 10^{-10} \xi_i \left(\frac{I_1^H}{I}\right)^2 T^{1/2}(°K)\, x\psi_1(x, \gamma_i)\ \mathrm{cm}^3\mathrm{sec}^{-1}. \tag{28}$$

The values of $\psi_1(x, \gamma_i)$ are given on Table, and ψ_1 can be approximated by

$$\psi_1(x, \gamma_i) \cong \left(\frac{e^{-x}}{1+x}\right)\left[\frac{1}{20+x} + \ell n\{1.25\gamma_i(1+\frac{1}{x})\}\right] \tag{29}$$

(3) Lotz (1967, 1968)

Lotz deduced a cross section from many experimental results

$$\sigma = \sum_{j=1}^{N} a_j \zeta_j \frac{\ln(E/I_j)}{EI_j} \{1 - b_j \exp[-c_j(E/I_j - 1)]\},$$

where I_j(ev) is the binding energy of an electron in j-th subshell (j = 1 means the outermost subshell), ζ_j is the number of equivalent electrons of j-subshell, and a_j, b_j, c_j are individual constants which have been determined by a reasonable guess. The values a_j, b_j, c_j are given in Tables for certain ions from hydrogen to Calcium by Lotz (1967) and from Scandium to Zinc by Lotz (1968). For the cross sections known experimentally, this formula gives accurate results within experimental error. For the ions ionized more than four times, he assumed that $a_i = 4.5 \times 10^{-14}$ cm^2 (ev)^2, $b_i = 0$ and $c_i = 0$ in order to have agreement with the theoretical calculations of Rudge and Schwartz (1966) for hydrogen-like ions, as there are no experimental data. He mentions that the validity of this assumption for ions not hydrogen-like might be questionable. The number N of subshells to consider was taken to be 2 for ions from H to Ca, and to be 3 from Sc to Zn.

The rate coefficient is

$$\beta = 6.7\times10^{7} \sum_{j=1}^{N} \frac{a_j \zeta_j}{T_e^{3/2}(\mathrm{ev})} \{ \frac{1}{I_j/T_e} \int_{I_j/T_e}^{\infty} \frac{\exp(-z)}{z}\, dz - \frac{b_j \exp C_j}{I_j/T_e + C_j} \int_{I_j/T_e + C_j}^{\infty} \frac{\exp(-y)}{y}\, dy \} \qquad (31)$$

He gave the cross section curves for atoms and for singly charged ions, and he estimated the error to be not higher than $^{+40}_{-30}$ %.

The calculated values of β are listed numerically in Table for all ionized states for the temperature of 1 ~ 10^4 eV. The error becomes large for temperatures lower than the ionization potential.

(4) Seaton (1964)

Seaton approximated $u\sigma^R$ as a straight line ($u \lesssim 2$) like

$$\sigma = 2.2 \left(\frac{I_H}{I}\right)^2 \zeta \frac{(u-1)}{u} \pi a_0^2 \tag{32}$$

and

$$\beta = 2.0\times10^{-8} T^{1/2}(°K) \Sigma \zeta_j(n,\ell) I_j^{-2}(n,\ell) e^{-\frac{I_j}{KT}} \text{cm}^3\text{sec}^{-1} \tag{33}$$

where ζ_j is the number of electrons in subshell (n, ℓ), I_j(ev) is the ionization energy. This formula is good for highly charged positive ions, and for low energies. Jordan (1969) used this formula to calculate the ionization equilibrium in a hot thin plasma for the elements from C to Ni.

(5) Hydrogen-like ions

Percival (1966) gave formulae to calculate the average cross section for ionization from excited state of hydrogen and hydrogenic ions. There is no limitation on the energy range of validity.

i) ionization from the ground state ($i = 1$) for hydrogen atom

$$\sigma_1^H = \frac{(1.19 \ln u_1 + 5.26)(u_1 - 1)}{u_1^2 + 1.67u_1 + 3.57} \pi a_o^2 \tag{34}$$

ii) ionization from the excited state (i > 1) for hydrogen atom

$$\sigma_{i>1}^H = \frac{(1.28i^{-1} \ln u_i + 6.67)(u_1 - 1)}{u_1^2 + 1.67u_1 + 3.57} \pi a_o^2 i^4 \tag{35}$$

iii) hydrogenic ions (charge number Z)

$$\sigma^z = \sigma_i^H [1 + \frac{2.3}{(1-Z^{-1})^2 + 2(u_1-1)^2}] Z^{-4} \tag{36}$$

It is necessary to do numerical calculations for getting the ionization rate coefficient β.

Jacobs (1972) gave a formula for β adjusting the numerical constants to the experimental results for Z = 1 and Z = 2 and to the calculated values for Z = 50.

$$\beta = 6.67\times10^{-9}(2.42 - \frac{1.72}{Z} + \frac{0.19}{Z^2})(\frac{n}{Z})^4 \sqrt{T_e(ev)}\, e^{-x} \tag{37}$$

III. Comparison

We show in Fig.4 the comparison between the values calculated by some empirical formulae discussed above, and the experimental results of Aitken and Harrison (1971), for the ionization cross section of OIII → OIV. The result by Seaton (1964) is good at low energies, but is too large at high energies. The formula by Lotz give underestimated results near threshold, but gives good agreement at high energies. The values by Post (1961) and by Drawin (1966) are smaller by a factor of 5 and 1.7 respectively than the experimental results.

We show in Fig.5 the results of various empirical formulae for the ionization rate coefficient β. As the approximation method for evaluating the integral depends on the empirical formula, the trend of the absolute value of β does not always coincide with that of σ. For example, the cross section by Post (1961) is small but his rate coefficient gives a value larger than any other result. The result calculated by Seaton (1964) is larger than that by Lotz.

The same comparison as in Fig.5 for the rate coefficient of FeXV → FeXVI are shown in Fig.6. Rudge and Schwartz (1966) calculated the ionization cross section and rate coefficient using the Bon-exchange approximation, and they are shown in Fig.6 with dotted line. The values by Seaton and by Lotz are both larger than the theoretical results in the high energy range.

The formulae (32) (33) by Seaton are simple and can be used easily, but they are not suitable at high energies ($u > 2$, $x > 2$). The formulae by Lotz are rather complicated but we can use the tabulated numerical results for rate coefficient.

Figure Captions

Fig.1 Integrated gaunt factor P(y) in eq.(17) which is proposed by Mewe (1972) to calculate electron collision excitation coefficient for the sequences other than H, He, Li and Ne-sequences.

$P(y) = A + (By - Cy^2 + D)e^y E_1(y) + Cy$

where y is the ratio of the excitation energy E_{ij} to the electron temperature kT.

(i) allowed transition $\Delta n \neq 0$

(ii) allowed transition $\Delta n = 0$

(iii) forbideen monopole or quadrupole transition

(iv) spin exchange transtion

Fig.2 Comparison of effective gaunt factor g(u) between several empirical formulae as a function of u. u is the ratio of the electron energy E to the excitation energy E_{ij}.

$\sigma = 1.28\times10^{-15}(I_H/E_{ij})^2 f_{ij} g(u) u^{-1}$

(1) Post (1961), (2) Van Regemorter (1962), (3) Drawin (1966), (4) Mewe (1972)

Fig.3 Excitation rate coefficient as a function of temperature.

(a) For the transitions of NeVII 465Å $2s^2(^1S) - 2s2p(^1P)$ and NeVII 895Å $2s^2(^1S) - 2s2p(^3P)$.

Post (1961), Van Regemorter (1962), Drawin (1966), Mewe (1972) ···· empirical formula.

Osterbeck (1970) ···· theory.

Johnston III and Kunze (1971) ···· experiment.

(b) For the transitions of OVII 21.6Å $1s^2(^1S)$ - $1s2p(^1P)$ and OVII 21.8Å $1s^2(^1S)$ - $1s2p(^3P)$.

Post (1961), Van Regemorter (1962), Drawin (1966), Mewe (1972) ···· empirical formula.

Elton and Köppendörfer (1967) ···· experiment

... $1^1S \rightarrow (2^1S+2^1P)$

... $1^1S \rightarrow (2^3S+2^3P)$

... $1^1S_0 \rightarrow 2^1P_1$

Fig.4 Ionization cross section for $e + O^{+2} \rightarrow e + O^{+3} + e$

Aitoken and Harrison (1971) ····· experiment.

Post (1961), Seaton (1964), Drawin (1966), Lotz (1967, 68) ····· empirical formula.

Fig.5 Ionization rate coefficient for $e + O^{+2} \rightarrow e + O^{+3} + e$

Post (1961), Seaton (1964), Lotz (1967, 68), Drawin (1966) ····· empirical formula.

Fig.6 Ionization rate coefficient for $e + Fe^{+14} \rightarrow e + Fe^{+15} + e$

Seaton (1964), Lotz (1967, 68) ····· empirical formula.

Rudge and Schwartz (1966) ····· theory.

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Table I

Coefficients A, B, C, D in the formula by Mewe

Isoel. seq.	Transition	Z	A	B	C	D	g(u=1)
H	1s-2s	2	0.08	-0.16	0.11	0	0.03
		>2	0.055	0	0	0	0.055
	1s-2p	2	0.04	0.04	0.02	0.28	0.10
		>2	0.04	0.21	-0.04	0.28	0.21
	1s-3	2	0.20	0.06	0	0.28	0.26
		>2	0.27	0.08	0	0.28	0.35
	1s-4	2	0.25	0.04	0	0.28	0.29
		>2	0.33	0.05	0	0.28	0.38
	1s-5	2	0.27	0.03	0	0.28	0.30
		>2	0.36	0.04	0	0.28	0.40
	1s-6	2	0.28	0.02	0	0.28	0.30
		>2	0.37	0.03	0	0.28	0.40
	1s-n(n≧7)	2	0.29	0.02	0	0.28	0.31
		>2	0.38	0.03	0	0.28	0.41
He	$1s^1S-2s^1S$	3	0.05	-0.04	0	0	0.01
		4~10	0.05	0	0	0	0.05
	$1s^1S-3s^1S$	3~10	0.05	0.01	0	0	0.06

Table I (continued 1)

Isoel seq.	Transition	Z	A	B	C	D	g(u=1)
He	$1s^1S-ns^1S$	>10	0.055	-0.02	0	0	0.035
	$1s^1S-2p^1P$	3	0.02	0.02	0	0.28	0.04
		4~10	0.04	0.2	-0.04	0.28	0.2
	$1s^1S-3p^1P$	3~10	0.2	0.05	0	0.28	0.25
	$1s^1S-np^1P$	>10	0.06	0.06	0	0.28	0.12
	$1s^1S-3d^1D$	≧3	0.02	0	0	0	0.02
	$1s^1S-n(1)$ $(n \geqq 4)$	≧3	0.3	0.05	0	0.28	0.35
	$1s^1S-2s^3S$	3~10	0	0	0.07	0	0.07
	$1s^1S-2p^3P$	3~10	0	0	0.2	0	0.2
	$1s^1S-np^3P$	>10	0	0	0.12	0	0.12
Li	2s-2p	≧4	0.7(1-0.5x)	1-0.8x	-0.5(1-x)	0.28	1.2(1-0.54x)
	2s-3p	4, 5	-0.3	0.6	-0.2	0.28	0.1
		>5	-0.32(1-0.9x)	0.88(1-1.7x)	-0.27(1-2.1x)	0.28	0.29(1-2.2x)
	2s-np $(n \geqq 4)$	4, 5	-0.26	0.63	-0.3	0.28	0.07
		>5	-0.16(1+2x)	0.8(1-0.7x)	-0.2	0.28	0.44(1-2x)
	2s-3s	≧4	0.11(1+3x)	0	0	0	0.11(1+3x)

Table I (continued 2)

Isoel. seq.	Transition	Z	A	B	C	D	g(u=1)
Li	2s-4s	4	0.17	0	0	0	0.17
		>4	0.13	0	0	0	0.13
	2s-5s	≦10	0.11	0	0	0	0.11
		>10	0.13	0	0	0	0.13
	2s-6s	≦10	0.1	0	0	0	0.1
		>10	0.14	0	0	0	0.14
	2s-ns(n≧7)	≦10	0.09	0	0	0	0.09
		>10	0.15	0	0	0	0.15
	2s-3d	≧4	0.35(1+2.7x)	-0.11(1+5.4x)	0	0	0.24(1+1.5x)
	2s4d	4	0.54	-0.25	0	0	0.29
		>4	0.31	-0.1	0	0	0.21
	2s-5d	4	0.43	-0.19	0	0	0.24
		>4	0.27	-0.08	0	0	0.19
	2s-6d	4	0.35	-0.15	0	0	0.2
		>4	0.24	-0.06	0	0	0.18
	2s-nd(n≧7)	4	0.3	-0.12	0	0	0.18
		>4	0.22	-0.05	0	0	0.17

Table I (continued 3)

Isoel. seq.	Transition	Z	A	B	C	D	g(u=1)
Ne	$2p^6$-$2p^5 3d$	$\geqq 11$	0.05	0.2	0	0.28	0.25
	$2p^6$-$2p^5 4d$	$\geqq 11$	0.2	0.15	0	0.28	0.35
	$2p^6$-$2p^5 3s$	<30	-0.17	0.25	0	0.28	0.08
		$\geqq 30$	-0.1	0.25	0	0.28	0.15
	$2p^6$-$2p^5 4s$	<30	-0.04	0.2	0	0.28	0.16
		$\geqq 30$	0.05	0.2	0	0.28	0.25
	$2s^2 2p^6$-$2s2p^6 3p$	<15	-0.3	0.4	0	0.28	0.1
		$\geqq 15$	-0.3	0.5	0	0.28	0.2
	$2s^2 2p^6$-$2s2p^6 4p$	<15	-0.2	0.3	0	0.28	0.1
		$\geqq 15$	-0.2	0.5	0	0.28	0.3

Z is the atomic number, $x = (Z-3)^{-1}$, n is the principal quantum number.

TABLE II

COMPARISON OF VALUES OF COLLISION STRENGTHS

Iso el. sequence	Transition	Ion	λ (Å)	Osci.stren. f	Ω_M	Ω	Ω_M/Ω
Be	$2s^2 - 2s2p$	NIV	765.1	0.64	5.9	$3.4^{(1)}$	1.7
						$5.7^{(2)}$	1.04
		NeVII	465.2	0.41	2.3	$1.84^{(1)}$	1.25
						$3.05^{(2)}$	0.75
	$2s^2 - 2s3p$	NIV	247.2	0.55	0.67	$0.23^{(1)}$	2.9
		OV	172.2	0.59	0.50	$0.14^{(1)}$	3.6
		NeVII	97.6	0.45	0.22	$0.054^{(1)}$	4.1
	$2s^2 - 2s3s$	NIV	257.1	(0.55)	0.34	$0.23^{(1)}$	1.5
		OV	178.2	(0.59)	0.25	$0.079^{(1)}$	3.1
		NeVII	100.2	(0.45)	0.108	$0.037^{(1)}$	2.9
	$2s^2 - 2s3d$	NIV	233.0	(0.55)	0.31	$0.23^{(1)}$	1.3
		OV	163.2	(0.59)	0.23	$0.26^{(1)}$	0.88
		NeVII	93.5	(0.45)	0.10	$0.07^{(1)}$	1.4
	$2s^2 - 2s4P$	NIV	197.2	0.13	0.126	$0.11^{(1)}$	1.1
		OV	135.5	0.138	0.092	$0.045^{(1)}$	2.0
	$2s^2 - 2s4d$	NIV	194.3	(0.13)	0.060	$0.15^{(1)}$	0.4
		OV	140.0	(0.138)	0.046	$0.04^{(1)}$	1.15
		NeVII	75.0	(0.165)	0.0295	$0.017^{(1)}$	1.7
	$2s^2 - 2s4f$	OV	133.3	(0.138)	0.044	$0.032^{(1)}$	1.4
	$2s^2 - 2s4s$	OV	136.7	(0.138)	0.045	$0.034^{(1)}$	1.3
Mg	$3s^2 - 3s3p$	FeXV	284.3	1.18	4.0	$3.37^{(3)}$	1.2
						$2.7^{(4)}$	1.5
	$3s^2 - 3s3d$	FeXV	137.1	(1.18)	0.38	$0.312^{(3)}$	1.2
						$0.18^{(4)}$	2.1

TABLF II (continued)

Iso el. sequence	Transition	Ion	λ (Å)	Osci.stren. f	Ω_M	Ω	Ω_M/Ω
Al	$3s^2 3p$ - $3s3p^2$	FeXIV	260.	0.53	9.84	10.2 (5)	0.96
	$3s^2 3p$ - $3s^2 3d$	"	211.3	0.63	10.6	10.3 (5)	1.03
	$3s^2 3p$ - $3s^2 4p$	"	95.2	(0.27)	0.37	0.45 (5)	0.82
	$3s^2 3p$ - $3s^2 4s$	"	71.5	0.062	0.12	0.047 (5)	2.5
	$3s^2 3p$ - $3s^2 4d$	"	59.0	0.273	0.46	0.12 (5)	3.8
	$3s^2 3p$ - $3s^2 4f$	"	56.4	(0.27)	0.22	0.47 (5)	0.47
Si	$3s^2 3p^2$ - $3s3p^3$	FeXIII	246.	0.22	5.9	6.4 (7)	0.92
	$3s^2 3p^2$ - $3p3d(^3D)$	"	201.	0.49	10.7	15.98 (7)	0.67
	$3s^2 3p^2$ - $3p3d(^1F)$	"	177.1	(0.49)	0.55	0.228 (7)	2.4
K	$3p^6 3d$ - $3p^6 4f$	FeVIII	131.	0.604	3.9	0.94 (6)	4.1
	3d - 4p	"	196	0.039	0.38	0.179 (6)	2.1
	3d - 5f	"	108	0.24	1.2	0.369 (6)	3.2
	3d - 6f	"	98.5	0.117	0.57	0.18 (6)	3.2
	3d - 7f	"	93.	0.034	0.16	0.103 (6)	1.6

Notes for Table I

(1) W. D. Johnston III and H. J. Kunze (1971), Physical Review A, 4, 962.

(2) D. E. Osterbrock (1970), J. Phys. B., 3, 149.

(3) O. Bely and M. Blaha (1968), Solar Physics 3, 563.

(4) D. R. Flower (1971), J. Phys. B., 4, 697.

(5) M. Blaha (1971), Solar Physics 17, 99.

(6) S. J. Czyzak and T. K. Drueger (1966) Ap. J. 144, 381.

(7) D. R. Flower and G. Pineau des Forêts (1973), Astron. and Astrophys. 24, 181.

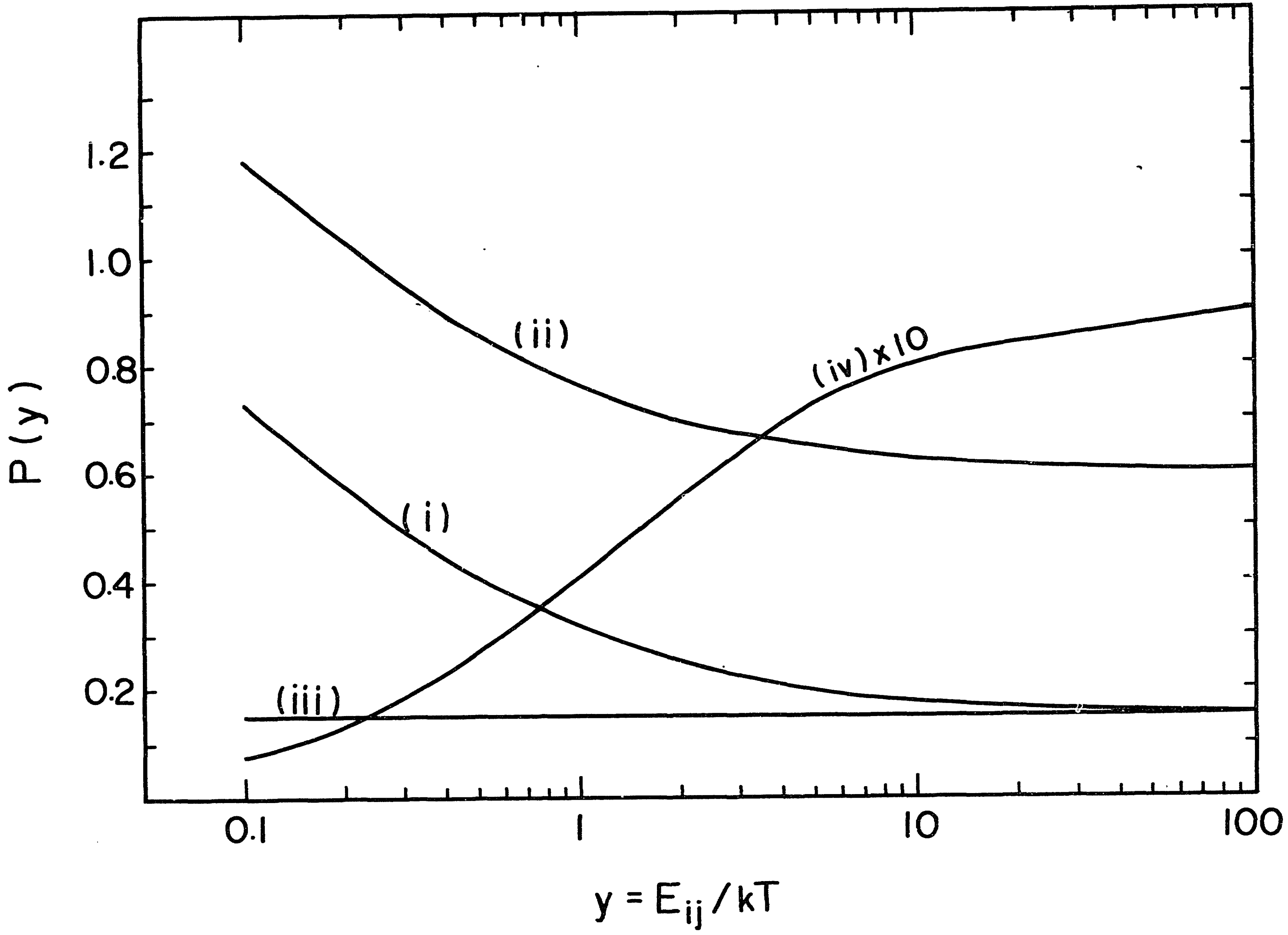
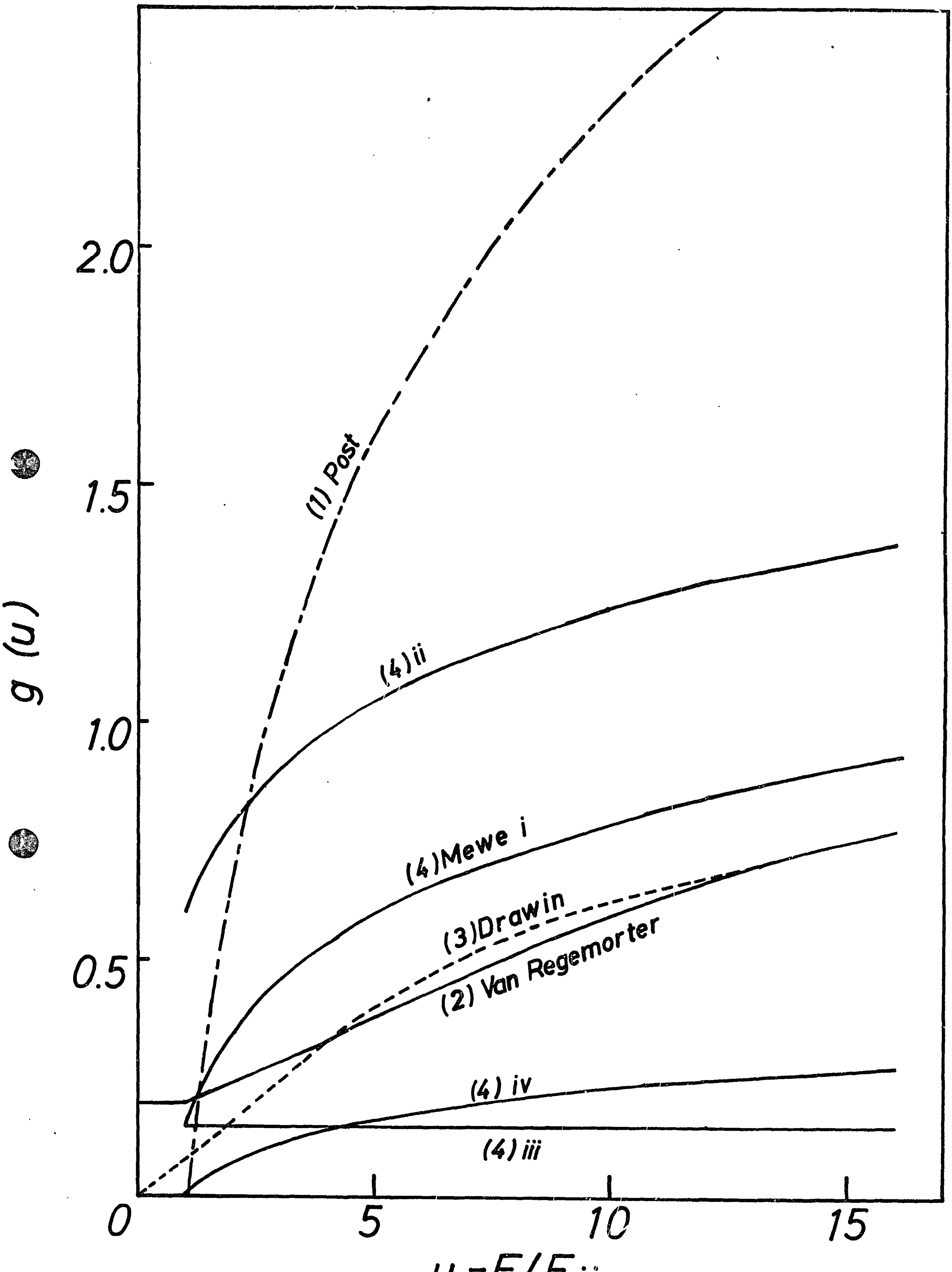
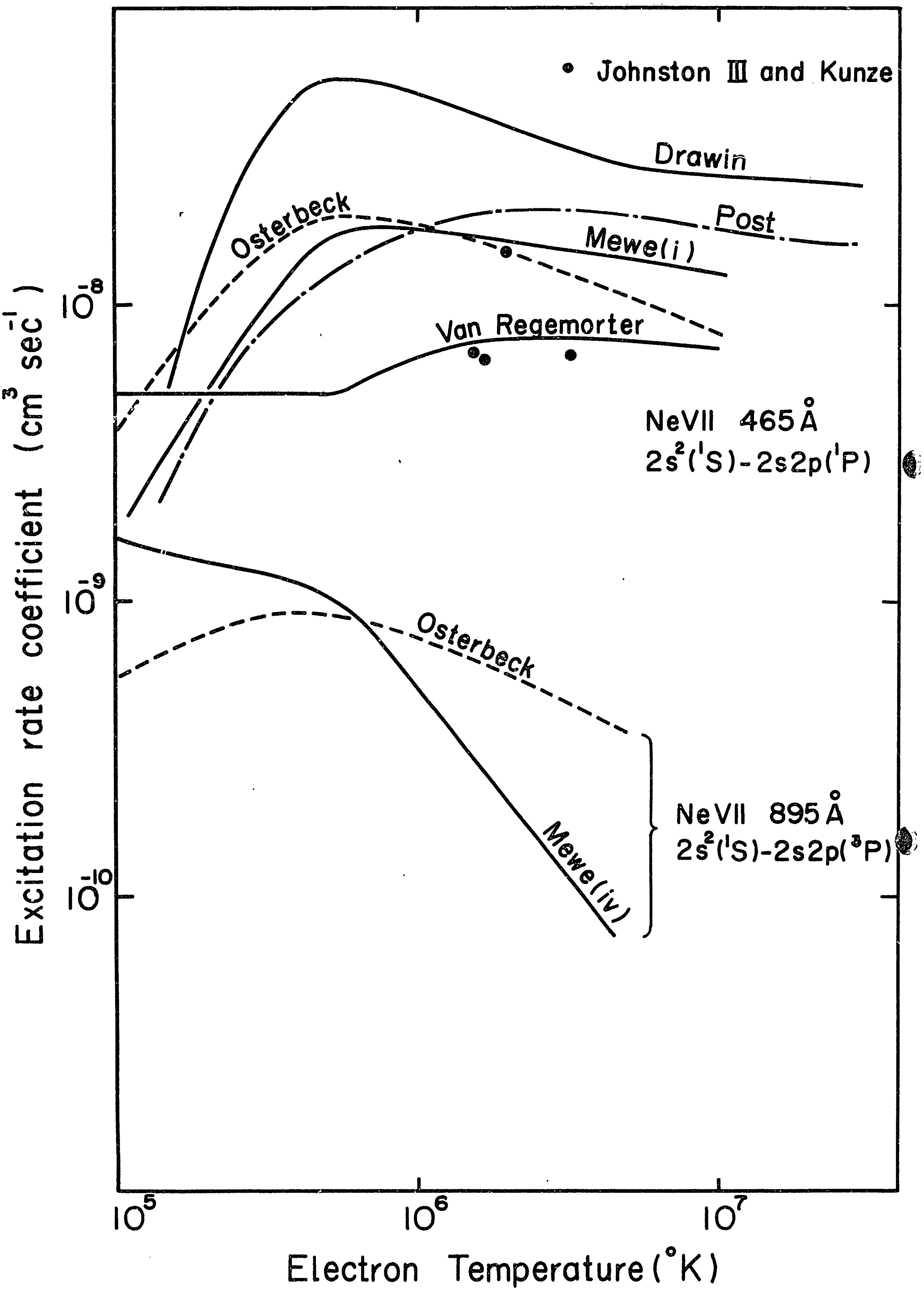


Fig. 1



g (u)
2.0
1.5
1.0
0.5
0
5
10
15
u = E/E_{ij}
(1) Post
(4) ii
(4) Mewe i
(3) Drawin
(2) Van Regemorter
(4) iv
(4) iii

Fig. 2



Johnston III and Kunze
Drawin
Post
Osterbeck
Mewe(i)
Van Regemorter
NeVII 465 Å
$2s^2(^1S)-2s2p(^1P)$
Osterbeck
Mewe(iv)
NeVII 895 Å
$2s^2(^1S)-2s2p(^3P)$
10^{-8}
10^{-9}
10^{-10}
10^5
10^6
10^7
Excitation rate coefficient ($cm^3 sec^{-1}$)
Electron Temperature(°K)

Fig. 3 (a)

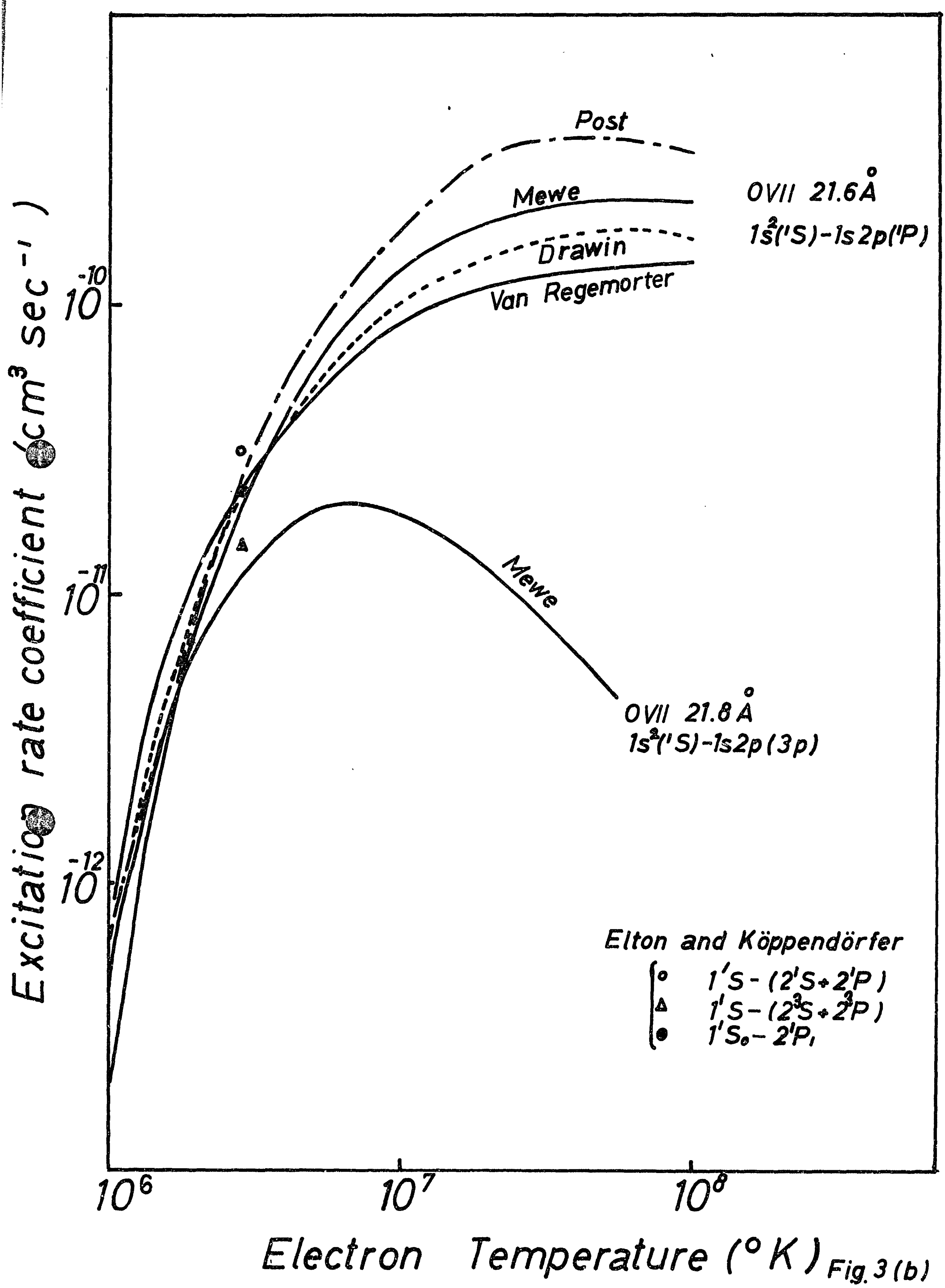

Post
Mewe
Drawin
Van Regemorter
OVII 21.6 Å
$1s^2(^1S)-1s2p(^1P)$
Mewe
OVII 21.8 Å
$1s^2(^1S)-1s2p(3p)$
Elton and Köppendörfer
○ $1^1S-(2^1S+2^1P)$
△ $1^1S-(2^3S+2^3P)$
● $1^1S_0-2^1P_1$
10^{-10}
10^{-11}
10^{-12}
10^6
10^7
10^8
Excitation rate coefficient ($cm^3\ sec^{-1}$)
Electron Temperature (°K)

Fig. 3 (b)

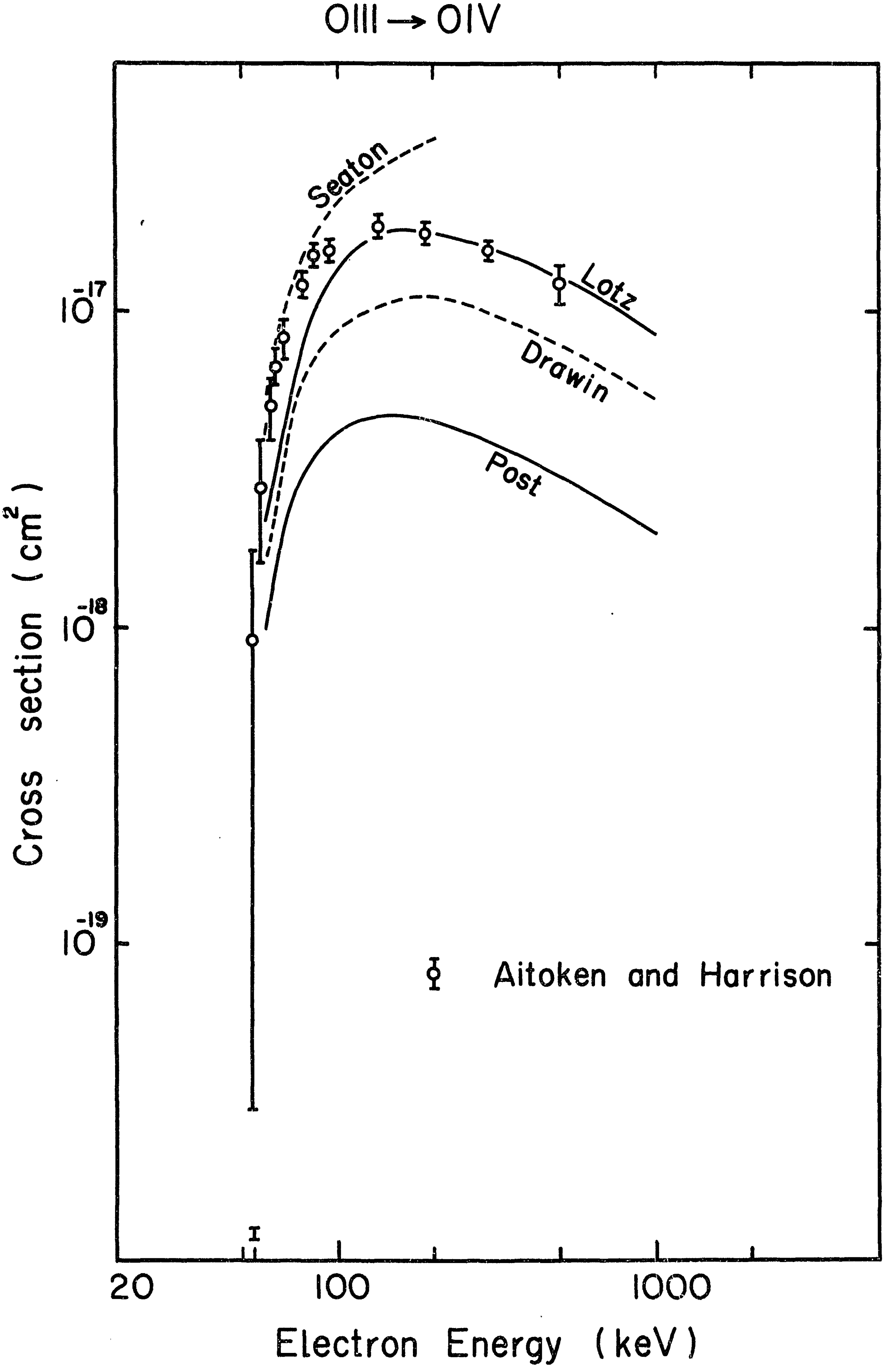

OIII → OIV
Seaton
Lotz
Drawin
Post
Aitoken and Harrison
Cross section (cm^2)
10^{-17}
10^{-18}
10^{-19}
20
100
1000
Electron Energy (keV)

Fig. 4

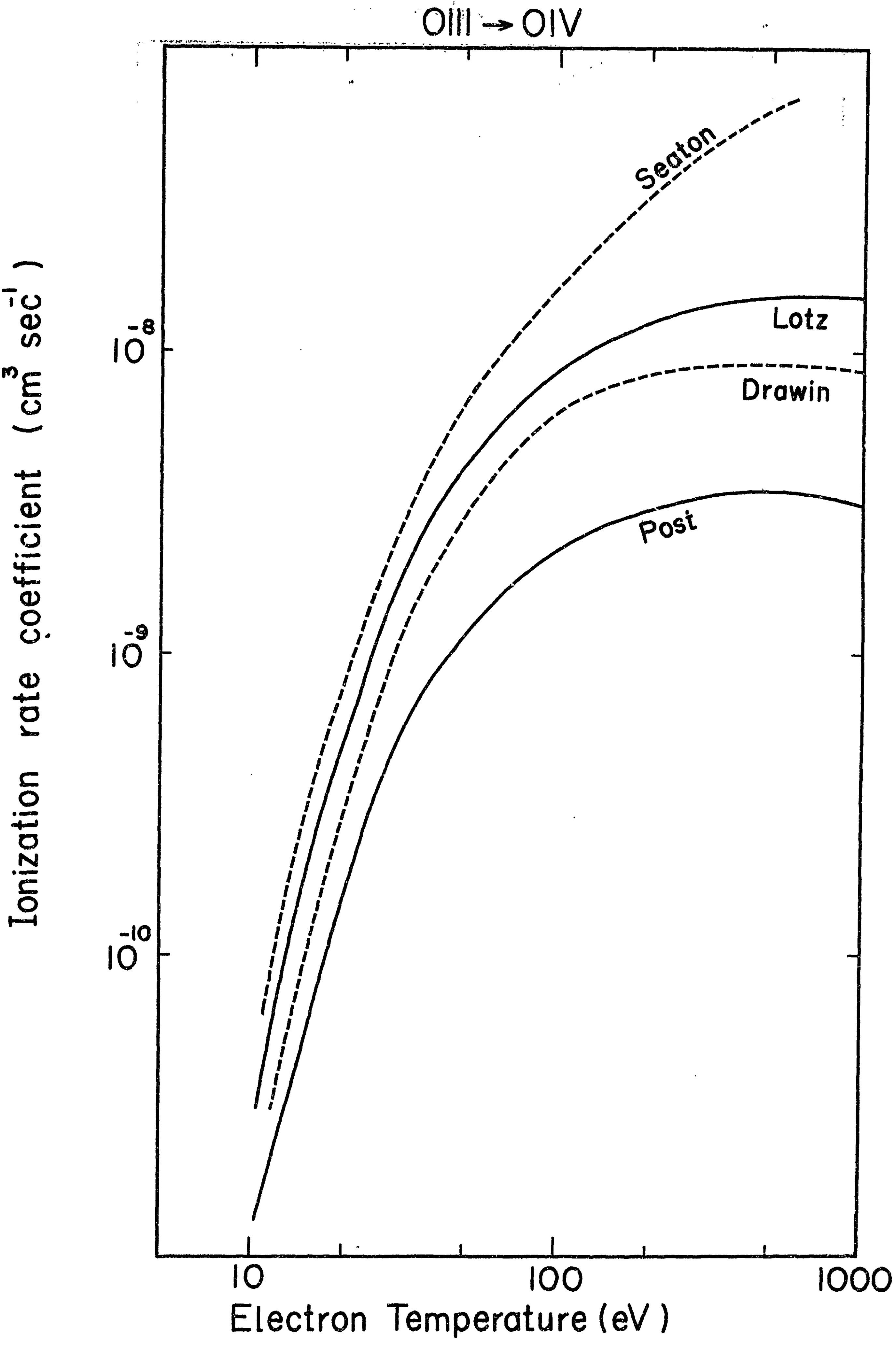

OIII → OIV
Seaton
Lotz
Drawin
Post
Ionization rate coefficient (cm^3 sec^{-1})
10^{-8}
10^{-9}
10^{-10}
10
100
1000
Electron Temperature (eV)

Fig. 5

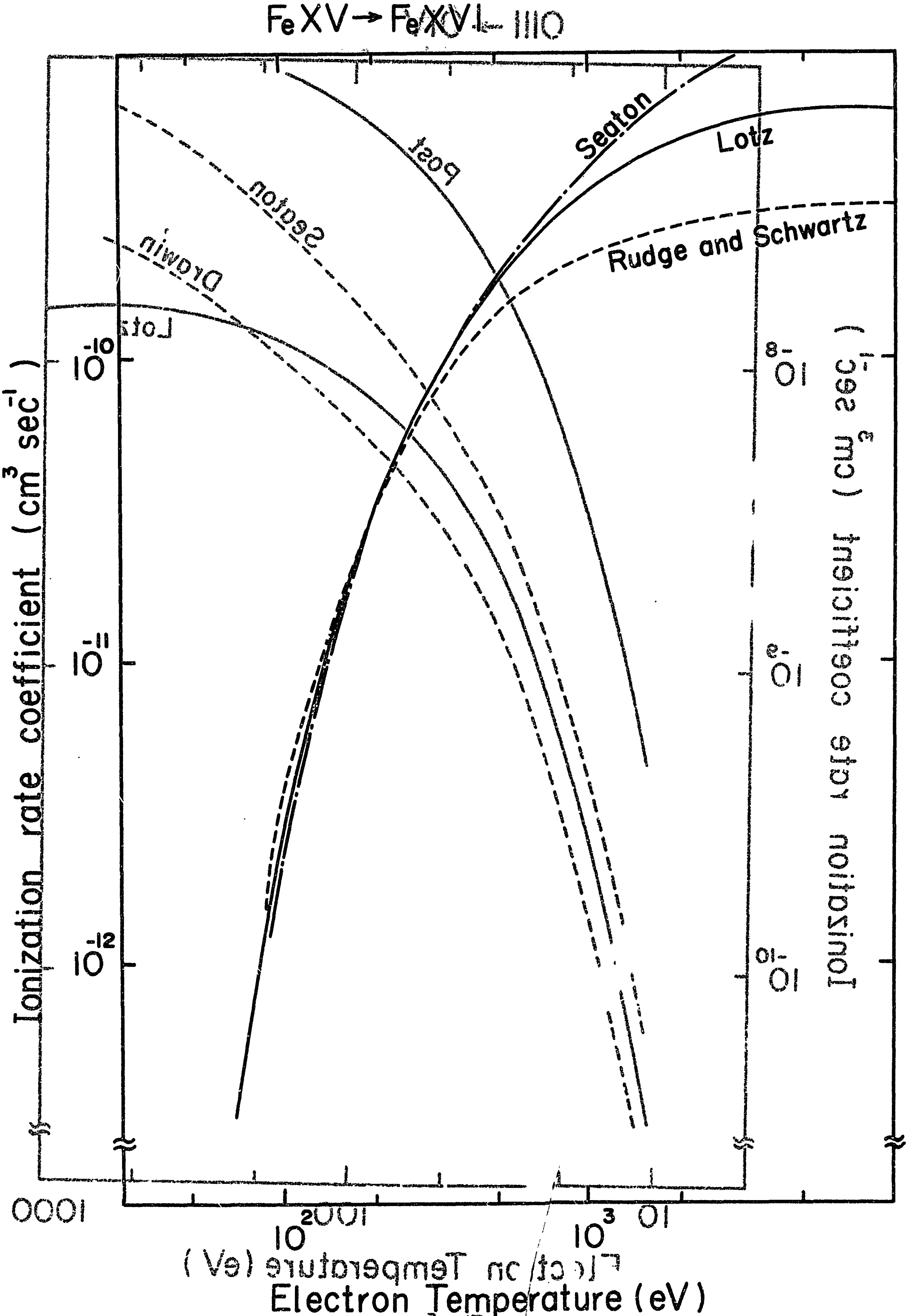

Fe XV → Fe XVI
Seaton
Lotz
Rudge and Schwartz
10^{-10}
10^{-11}
10^{-12}
Ionization rate coefficient (cm^3 sec^{-1})
10^2
10^3
Electron Temperature (eV)

Fig. 6