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FREE-FREE TRANSITION IN A PLASMA
—REVIEW OF CROSS SECTIONS AND SPECTRA—

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Abstract

The cross sections for free-free emission and free-free absorption were reviewed. The energy spectrum by free-free emission from a plasma of temperature Te is given as well as total emission rate. Free-free absorption coefficients are also given.

Free-Free Transition in a Plasma

Review of cross sections and spectra

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§1. Bremsstrahlung

Bremsstrahlung is emitted by a particle accelerated in the Coulomb field of an ion. It is also called free-free transition.

Several formulae are available in order to calculate the cross section of bremsstrahlung, depending on the energy range of the incident electron and on the emitted photon. We can divide the energy of the incident electron into two ranges as follows:

(1) relativistic region

(2) non-relativistic region

Roughly speaking we can use the Born approximation formula for the relativistic case, the Sommerfeld formula for the non-relativistic case , and classical theory for low energies ($W_0 < 13.6\ Z^2$eV). In the temperature range ($\sim$ 10 keV) which is of current interest in nuclear fusion, the Sommerfeld formula is suitable rather than the Born approximation. We consider these cases in the next sections.

Electron-electron bremsstrahlung in the relativistic region is investigated by Kawai et al (1978), and total emission rate is shown in §1. III (Fig.7).

The following symbols and units are used:

E_0, E;	initial and final total energy of the electron in a collision (in eV).	
W_0, W;	initial and final kinetic energy of the electron in a collision (in eV).	
λ ;	wavelength of the emitted photon (in Å).	
k_ν ;	energy of the emitted photon (in eV).	
kT_e ;	electron temperature (in eV).	
α ;	fine structure constant	1/137.043.
r_e ;	classical electron radius	2.82×10^{-13} cm.
m_e ;	electron mass	9.11×10^{-28} g.
η_0, η;	Sommerfeld number $\eta_0 = Z(\varepsilon_H/W_0)^{1/2}$, $\eta = Z(\varepsilon_H/W)^{1/2}$.	
ε_H ;	ionization energy of hydrogen atom	13.6 eV.
Z ;	nuclear charge	

I. Cross Section

(1) Relativistic case

For $\eta_0 \ll 1$ and $\eta \ll 1$, the Born approximation gives a good result. The cross sections at relativistic energies are reviewed by Koch and Motz (1959) and we refer to this paper for more detail about the differential and integrated cross sections.

From the Bethe-Heitler formula, the energy differential cross section $d\sigma$ is written as

$$d\sigma = 4Z^2\alpha r_e{}^2 \frac{dk_\nu}{k_\nu} f(E_0, E)$$

$$= 2.34 \times 10^{-27} Z^2 \frac{dk_\nu}{k_\nu} f(E_0, E) \quad cm^2 \quad . \tag{1}$$

Function $f(E_0, E)$ depends on the value of ξ which characterizes the screening effect and

$$\xi \equiv 100 \left(\frac{k_\nu m_e c^2}{E_0 \; E \; Z^{1/3}}\right) = \frac{5.11 \times 10^7 k_\nu}{(E_0 E \; Z^{1/3})}$$

$$= \frac{6.3 \times 10^{11}}{E_0 E \lambda Z^{1/3}} \quad ,$$

where λ is the wavelength of the photon. The screening effect by electrons of an ion (atom) is effective for the case of extreme relativistic and low energies. The Coulomb correction becomes effective for $Z > 20$ ($\sim$1% for $Z = 92$, e.g. Hayakawa 1969) in the case of no screening, whereas it is negligible in the case of complete screening.

(a) no screening $\xi >> 1$ (E_0, E, $k_\nu >> 5 \times 10^5$ eV)

$$f(E_0, E) = [1+(\frac{E}{E_0})^2 - \frac{2}{3}(\frac{E}{E_0})][\ln(\frac{2E_0 E}{k_\nu m_e c^2}) - \frac{1}{2}]$$

$$= [1+ (\frac{E}{E_0})^2 - \frac{2}{3}(\frac{E}{E_0})][\ln(\frac{E_0 E}{2.56 \times 10^5 \; k_\nu}) - \frac{1}{2}]$$

$$= [1+ (\frac{E}{E_0})^2 - \frac{2}{3}(\frac{E}{E_0})][\ln(\frac{E_0 E \; \lambda}{3.17 \times 10^9}) - \frac{1}{2}] \quad . \tag{2}$$

Z^2 in eq.(1) can be replaced by $(Z^2 + Z_e)$ in this case, where Z_e is the number of electrons per ion.

(b) Intermediate screening I (15 > ξ > 2)

$$f(E_0, E) = [1+ (\frac{E}{E_0})^2 - \frac{2}{3}(\frac{E}{E_0})][\ln(\frac{E_0 E}{2.56 \times 10^5 k_\nu}) - \frac{1}{2} - C(\xi)]$$

$$= [1+ (\frac{E}{E_0})^2 - \frac{2}{3}(\frac{E}{E_0})][\ln(\frac{E_0 E\ \lambda}{3.17 \times 10^9}) - \frac{1}{2} - C(\xi)]. \tag{3}$$

The values of $C(\xi)$ are given in Fig.1.

(c) Intermediate screening II (ξ < 2)

$$f(E_0, E) = (1+(\frac{E}{E_0})^2)[\frac{\phi_1(\xi)}{4} - \frac{1}{3}\ln Z] - \frac{2}{3}\frac{E}{E_0}[\frac{\phi_2(\xi)}{4} - \frac{1}{3}\ln Z]. \tag{4}$$

(d) Complete screening (ξ = 0)

$$f(E_0, E) = [1+(\frac{E}{E_0})^2 - \frac{2}{3}(\frac{E}{E_0})]\ln(183\ Z^{-1/3}) + \frac{1}{9}\frac{E}{E_0}\ . \tag{5}$$

Gould (1969) has calculated $f(E_0, E)$ for the hydrogen atom, the helium ion and the helium atom, and the results are shown in Vol.2 of Cross Sections for Atomic Process (1976).

The spectral cross section for Platinum (Z = 78) is shown in Fig.3 as an example (Koch and Motz, 1959).

(2) Non-relativistic case

The cross section is usually expressed by means of the Gaunt factor g_{ff} and the classical cross section $d\sigma_{class}$ given by Kramers (1923),

$$d\sigma = d\sigma_{class}\, g_{ff}$$

$$= \frac{16}{3}\frac{\pi}{\sqrt{3}}\alpha r_e^2 \frac{m_e c^2}{2W_0} Z^2 \frac{dk_\nu}{k_\nu} g_{ff}$$

$$= 1.43 \times 10^{-21} \frac{1}{W_0} g_{ff} \frac{dk_\nu}{k_\nu}. \qquad (6)$$

The free-free Gaunt factor is in general a function of electron energy and photon energy.

(a) Born approximation

The Born approximation is valid for high energy incident electrons ($\eta_0 << 1$, $\eta << 1$) and for the low energy spectrum. The following formula is known as the Born formula, and is suitable only for the long-wavelength limit and the range of $0.33 < \eta < 0.22$.

$$g_{ff} = \frac{\sqrt{3}}{\pi} \ln \frac{\eta+\eta_0}{\eta-\eta_0} \quad . \qquad (7)$$

(b) Sommerfeld formula

Sommerfeld (1939) derived an exact quantum mechanical expression in terms of the hypergeometric function using the non-relativistic dipole approximation and neglecting the recoil of the ion.

$$g_{ff} = \frac{\sqrt{3}\pi\, x(d/dx)\{|F(i\eta_0, i\eta, 1:x)|^2\}}{(e^{2\pi\eta_0} - 1)(1 - e^{2\pi\eta})} \quad , \qquad (8)$$

where

$$x = - 4\eta_0\eta/(\eta - \eta_0)^2 \quad ,$$

and $F(i\eta_0, i\eta, 1:x)$ is the hypergeometric function. This formula is not suitable for the relativistic case, since higher order multipole radiation is not considered.

The expression of eq.(8) is exact but difficult to evaluate because of the hypergeometric function. Brussaard and Van de Hulst (1962) collect many approximations that permit the rapid calculation of numerical results with an accuracy of one percent.

Karzas and Latter (1961) calculated numerically the exact values of g_{ff} from eq.(8) for a wide range of incident electron energy W_0 and the emitted photon energy k_ν. Those are shown in Fig.4.

(c) Born-Elwert formula

From eq.(8) Elwert derived a formula which is fairly accurate over the whole range of the electron and photon energies,

$$g_{ff} = \frac{\sqrt{3}}{\pi} \ln \frac{\eta+\eta_0}{\eta-\eta_0} \left(\frac{\eta}{\eta_0} \frac{1 - e^{-2\pi\eta_0}}{1 - e^{-2\pi\eta}}\right) \quad . \tag{9}$$

This formula has an accuracy within 1% for $0.03 < \eta_0 < 0.22$.

(d) Classical theory

The classical result is valid only for low electron energies $\eta_0 >> 1$.

The Gaunt factor is expressed by the impact parameter method as follows

$$\begin{aligned} g_{ff} &= \frac{\sqrt{3}}{\pi} \ln \left(4\sqrt{2}\, h\, W_0{}^{3/2}/\Gamma\, Ze^2 m_e{}^{1/2} k_\nu\right) \qquad \text{for } k_\nu << T_0 \\ &= \frac{\sqrt{3}}{\pi} \ln \left(0.608\, W_0{}^{3/2} / Z \cdot k_\nu\right) \\ &= \frac{\sqrt{3}}{\pi} \ln \left(4.91 \times 10^{-5}\, W_0{}^{3/2} \lambda / Z\right) \quad , \end{aligned} \tag{10}$$

Where $\Gamma = e^\gamma = 1.781 \cdots$ with $\gamma = 0.577$ (Euler's constant).

$$g_{ff} = 1 \qquad \text{for } k_\nu >> T_0 \quad .$$

When ions are fully ionized, the Sommerfeld formula (8) can be used for low energies. Elwert (1948) showed that the classical formula (10) can be obtained from this formula for low electron energies.

(e) Screening effect

The results mentioned in (a), (b), (c) and (d) are solutions for a point charge Z, and screening effects by electrons in an ion are neglected.

For fully ionized ions, the effective charge Z_{eff} is equal to the nuclear charge Z, but generally it is complicated for other ions and atoms. Screening effects for ions and atoms are not yet well studied. (e.g. Guggenberger (1957), Hettner (1958)). The value of Z_{eff} depends on the photon energies and changes from the net charge $Z_i = Z - Z_e$ for low photon energies to the nuclear charge Z at high photon energies.

Near the high energy cut-off, the contribution from the minimum impact parameter P_{min} is dominant, and P_{min} is roughly equal to the de Broglie wavelength $h/m_e v_0 = 12.26/\sqrt{W_0(eV)}\,\text{Å}$ (Weinstock 1942). So we can consider that $Z_{eff} = Z$ if the de Broglie wavelength is smaller than the radius of K-shell, and $Z_{eff} = Z - 2$ if the de Broglie wavelength is between the L-shell and the K-shell, etc.

II. Temperature averaged spectrum for a Maxwellian gas

The energy spectrum by free-free emission from electrons which have a Maxwellian distribution can be written as

$$dP = N_e N_i \int_0^\infty f(v)\, d\sigma v k_\nu dv \qquad (11)$$

where $f(v)$ is Maxwellian distribution function $f(v)dv = 4\pi\left(\frac{m_e}{2\pi kT_e}\right)^{3/2} v^2 e^{-\frac{m_e v^2}{2kT_e}} dv$, N_i and N_e are ion and electron density respectively. The spectrum is

$$dP = \frac{2^5 e^6 \pi}{3\sqrt{3} m_e^2 c^3 h} \left(\frac{2\pi m_e}{kT_e}\right)^{1/2} Z_{eff}^2 N_e N_i \bar{g}_{ff}(u, \gamma^2) e^{-k_\nu/kT_e} dk_\nu$$

$$\frac{dP}{dk_\nu \cdot k_\nu} = \frac{9.53\times10^{-11}}{k_\nu (kT_e)^{1/2}} N_e N_i Z_{eff}^2 \bar{g}_{ff}(u, \gamma^2) e^{-k_\nu/kT_e}$$

$$\text{photons}(\text{cm}^3 \text{sec keV})^{-1}. \qquad (12)$$

or

$$\frac{dP}{d\lambda} = \frac{1.89\times10^{-21}}{\lambda^2 (kT_e)^{1/2}} N_e N_i Z_{eff}^2 \bar{g}_{ff}(u, \gamma^2) e^{-\frac{12400}{kT_e\, \lambda}} \text{erg}(\text{cm}^2 \text{sec Å})^{-1}$$

where $\bar{g}_{ff}(u, \gamma^2)$ is the temperature averaged Gaunt factor

$$\bar{g}_{ff}(u, \gamma^2) = \int_0^\infty e^{-x} g_{ff}\, dx \qquad (x^2 = Z_{eff}^2 Ry/kT_e). \qquad (13)$$

$\bar{g}_{ff}$ is a function of $u = k_\nu/kT_e$ and $\gamma^2 = Z_{eff}^2 Ry/kT_e$, and their values are calculated numerically by Karzas and Latter (1961). They are shown in Fig.5. Simple approximated analytic expressions for $\bar{g}_{ff}$ derived from the results by Karzas and Latters (1961) are given in Table I. For the case of Born approximation, starting from eq.(7), we find

$$\bar{g}_{ff} = \frac{\pi}{\sqrt{3}} e^{\frac{k_\nu}{2kT_e}} K_0\left(\frac{k_\nu}{2kT_e}\right)$$

$$= \frac{\pi}{\sqrt{3}} e^{\frac{6200}{\lambda \cdot kT_e}} K_0\left(\frac{6200}{\lambda \cdot kT_e}\right) \qquad (14)$$

where K_0 is the modified Bessel function. This function can be approximated as

$$K_0(k_\nu/2kT_e) = \ln(4kT_e/k_\nu) - 0.577 \text{ for } k_\nu \ll kT_e$$

$$K_0(k_\nu/2kT_e) = (\pi/2)^{1/2}\, e^{-k_\nu/2kT_e}\, (k_\nu/2kT_e)^{-1/2} \text{ for } k_\nu >> kT_e. \quad (15)$$

But formula (14) is not suitable for large values of γ^2 ($kT_e < 136\ Z_{eff}^2$ eV).

III. Total emission rate

The energy generation rate from a plasma is obtained by integrating eq.(12) over all photon energies.

$$U = \int_0^\infty \frac{dP}{dk_\nu}\, dk_\nu \ .$$

(1) Relativistic ($kT_e >> 5 \times 10^5$ eV) case

$$U = \frac{32}{3}\sqrt{\frac{2}{\pi}}\, N_i N_e Z^2 \alpha r_e^2 c \cdot kT_e \{2.28 \times 1.41 \ln (kT_e/m_e c^2)\} \quad (16)$$

$$= 3.35 \times 10^{-28} (\ln kT_e - 11.51) kT_e\, N_i N_e Z^2 \text{ erg}(\text{cm}^3\text{sec})^{-1}.$$

(2) Non-relativestic ($kT_e << 5 \times 10^5$eV) case

$$U = 1.53 \times 10^{-25} (kT_e)^{1/2} \langle g_{ff}(\gamma^2)\rangle\, N_i N_e Z_{eff}^2 \text{ erg}(\text{cm}^3\text{sec})^{-1} \quad (17)$$

where $\langle g_{ff}(\gamma^2)\rangle = \int_0^\infty du\, e^{-u}\, \bar{g}_{ff}(u, \gamma^2)$.

The values of $\langle g_{ff}(\gamma^2)\rangle$ are shown in Fig.6 $\langle g_{ff}(\gamma^2)\rangle$ takes the value between 1.15 and 1.45 for the temperature range of $kT_e < 13.6 \times 10^3 Z_{eff}^2$ eV.

(3) Electron-Electron bremsstrahlung

The total emission rate for e-e bremsstrahlung in the relativistic plasma is shown in Fig.7 (Kawai et al 1978).

§2. Free-Free absorption

Free-free absorption is the inverse process of bremsstrahlung, that is, an electron absorbs a photon and makes a transition to a higher state instead of to a lower state. This process is dominant for the absorption of photons of energies smaller than the photoionization threshold, and is important for low energy photons for the objects like thermal radio sources.

I. Cross section

The cross section for absorption of a photon $h\nu$ by an electron of the energy W_i, which makes a transition to energy $W_f = W_i + k_\nu$ in the Coulomb field, is related to the bremsstrahlung cross section σ (see §1, I, $W_i = W - k_\nu$, $W_f = W_0$), using the detailed balancing.

$$\rho = \frac{c^2 h^3 W_f}{4\pi\sqrt{2mW_i}} \frac{d\sigma}{k_\nu{}^2 dk_\nu} \quad . \tag{18}$$

It is convenient to use the free-free absorption coefficient τ^{FF} which is proportional to both the electron and ion densities

$$\tau^{FF} = N_i N_e \, \rho^{FF} \ \mathrm{cm}^{-1} \quad , \tag{19}$$

and ρ^{FF} is frequently called the free-free cross section, although it has the dimension of (length)^5.

(a) Hydrogenic transitions

Radiative transitions in the field of a point nuclear charge Z are called hydrogenic transitions.

The classical cross section for free-free absorption was first given by Kramers.

$$\rho_K^{FF} = \frac{4\pi h^2 e^6 Z^2}{3\sqrt{6}\, m_e^{3/2} c\, k_\nu^3 \sqrt{W_i}} = \frac{2}{3\sqrt{6}} \alpha r_e^2 \frac{\sqrt{m_e}\, h^3 c^4}{k_\nu^3 \sqrt{W_i}}$$

$$= 2.16 \times 10^{-37} Z^2 / k_\nu^3 (\mathrm{eV}) \sqrt{W_i (\mathrm{eV})} \quad \mathrm{cm}^5 \tag{20}$$

$$= 1.13 \times 10^{-49} Z^2 \lambda^3 (\text{Å}) / \sqrt{W_i (\mathrm{eV})} \quad \mathrm{cm}^5 \;.$$

The cross section calculated by quantum theory can be written in terms of the Gaunt factor as discussed in §1.

$$\rho^{FF} = \rho_K^{FF} \cdot g_{ff}(W_i, k_\nu) \quad .$$

The values of g_{ff} are the same as shown in Fig.4, but with a change of the parameters $W_0 - k_\nu \rightarrow W_i$, where W_i is the incident electron energy in the case of free-free absorption whereas W_0 is the incident electron energy in the case of bremsstrahlung.

(b) Relation to elastic scattering

For low energy photons, the free-free absorption cross section may be approximately expressed in terms of elastic scattering cross section. The cross section is given in the one electron approximation by (e.g. Johnston 1967)

$$\rho^{FF} = \frac{2\sqrt{2}}{3} \frac{h^2 e^2 \sqrt{W_f}\, \bar{W}}{m_e^{3/2} k_\nu^3 c} Q_d(\bar{W}) + O\left(\frac{k}{\bar{W}}\right)^2$$

$$\cong 5.71 \times 10^{-24} \sqrt{W_f}\, \bar{W}\, Q_d(\bar{W}) / k_\nu^3 \quad \mathrm{cm}^5 \tag{21}$$

$$= 2.99 \times 10^{-36} \sqrt{W_f}\, \bar{W}\, \lambda^3 Q_d(\bar{W}) \quad \mathrm{cm}^5 \;,$$

where $\bar{W}$ is the mean energy $\bar{W} \equiv \frac{1}{2}(W_i + W_f)$ and $Q_d(\bar{W})$ is the momentum transfer cross section (in cm^2). For non-hydrogenic ions, the cross section can be written by (Dalgarno and Lane

1966)

$$\rho^{FF} = 0.909 \times 10^{-24} \frac{W_i \sqrt{W_f}}{k^3} [\frac{W_f}{W_i} Q_d(W_i) + Q_d(W_f)] \text{ cm}^5 . \tag{22}$$

The data of the momentum transfer cross sections $Q_d(W)$ will be compiled in another section (Cross Section of Atomic Process Vol.3, 1978).

II. Absorption coefficient

Free-Free absorption coefficient μ is connected with the free-free emission rate P(eq.(12)) by Kirchhoff's law

$$\frac{P}{4\pi} = \mu B \quad , \tag{23}$$

where B is the Planck function

$$B_{k_\nu} = \frac{2k_\nu{}^3}{c^2 h^3} [\exp (\frac{k_\nu}{kT_e}) - 1]^{-1} \, dk_\nu$$

$$= 3.14 \times 10^{22} k_\nu{}^3 [\exp(\frac{k_\nu}{kT_e}) - 1]^{-1} \text{erg}(\text{cm}^2 \cdot \text{sec} \cdot \text{sr} \cdot \text{keV})^{-1}$$

$$B_\lambda = \frac{2hc^2}{\lambda^5} [\exp (\frac{hc}{\lambda kT_e}) - 1]^{-1} \, d\lambda \tag{24}$$

$$= 1.19 \times 10^{27} \lambda^{-5} [\exp(\frac{12400}{\lambda \cdot kT_e}) - 1]^{-1} \text{erg}(\text{cm}^2 \cdot \text{sec} \cdot \text{sr} \cdot \mathring{A})^{-1}$$

μ can be obtained from eq.(12) and eq.(23) for a Maxwellian electron gas, with a temperature averaged Gaunt factor $\bar{g}_{ff}$,

$$\mu = 1.27 \times 10^{-49} \lambda^3 (kT_e)^{-1/2} (1-e^{-\frac{12400}{\lambda \cdot kT_e}}) Z^2_{eff} N_e N_i \bar{g}_{ff} \text{cm}^{-1}$$

$$= 2.42 \times 10^{-37} k_\nu{}^{-3} (kT_e)^{-1/2} (1-e^{-\frac{k_\nu}{kT_e}}) Z^2_{eff} N_e N_i \bar{g}_{ff} \text{cm}^{-1} . \tag{25}$$

For low frequencies ($k_\nu << kT_e$)

$$\mu = 1.58 \times 10^{-45} \lambda^2 (kT_e)^{-3/2} Z^2_{eff} N_e N_i \bar{g}_{ff} \quad \text{cm}^{-1} \tag{26}$$

$$= 2.42 \times 10^{-37} k_\nu^{2} (kT_e)^{-3/2} Z_{eff}^2 N_e N_i \bar{g}_{ff} \quad cm^{-1} .$$

The values of $\bar{g}_{ff}$ are the same as those in Fig.5 and in Table I of §1. Sometimes the free-free absorption coefficient is expressed as

$$a = \mu/N_i P_e \quad (cm^4/dyne) \quad , \tag{27}$$

where $P_e = N_e kT_e$ (dyne cm^{-2}) is the electron pressure.

Peach (1967) calculated the free-free absorption coefficients for non-hydrogenic ions of H^+, He^+, He^{++}, Li^+, C^+, N^+, O^+, Na^+, Mg^+, Mg^{++}, Al^+, Si^+, K^+ and Ca^{++} for the temperature range T_e = 4000 ∿ 13000°K, and the results for O^+, K^+, Mg^{++} and Ca^{++} are shown in Fig.8. Dalgarno and Lane (1966) calculated for H, He, N, O, Ne, H_2, N_2 and O_2 by the use of the momentum transfer cross section. Some of the results are shown in Fig.9, where the electron temperature is 6300°K. De Vore (1965) calculated μ for N and N^+ in the wavelength range of 5 ∿ 500μ. The coefficient for atoms is typically much smaller by several orders of magnitude than for ions because of the short range of the electron-atom interaction relative to the Coulomb force.

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Table I Gaunt factor $\bar{g}_{ff}$ for bremsstrahlung

kT_e/Z^2_{eff} (eV)	$u = k_\nu/kT_e$	$\bar{g}_{ff}(u)$
$1.36 \sim 13.6$	< 0.5	$1.12\ u^{-1/6}$
	> 0.5	~ 1
$13.6 \sim 136$	< 0.1	$u^{-0.19}$
	> 0.1	$1.2\ u^{-0.1}$
$136 \sim 1360$	< 0.1	$\pi/\sqrt{3}\ K_0(\frac{u}{2})\ e^{u/2}$
	> 0.1	$u^{-0.3}$
$1360 \sim 13600$	all range	$\pi/\sqrt{3}\ K_0(\frac{u}{2})\ e^{u/2}$

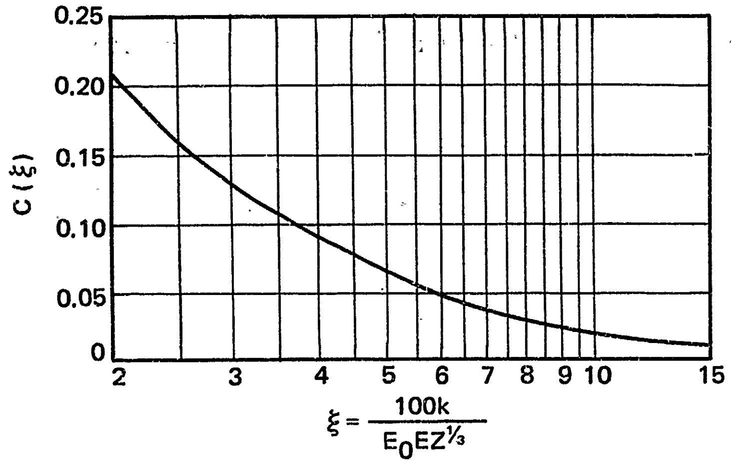

Fig. 1 Screening factor C (ξ) for the case of intermediate screening I (Eq. (3)) as a function of $\xi = 100k_\nu/E_0EZ^{1/3}$. (from Bethe and Heitler, 1934)

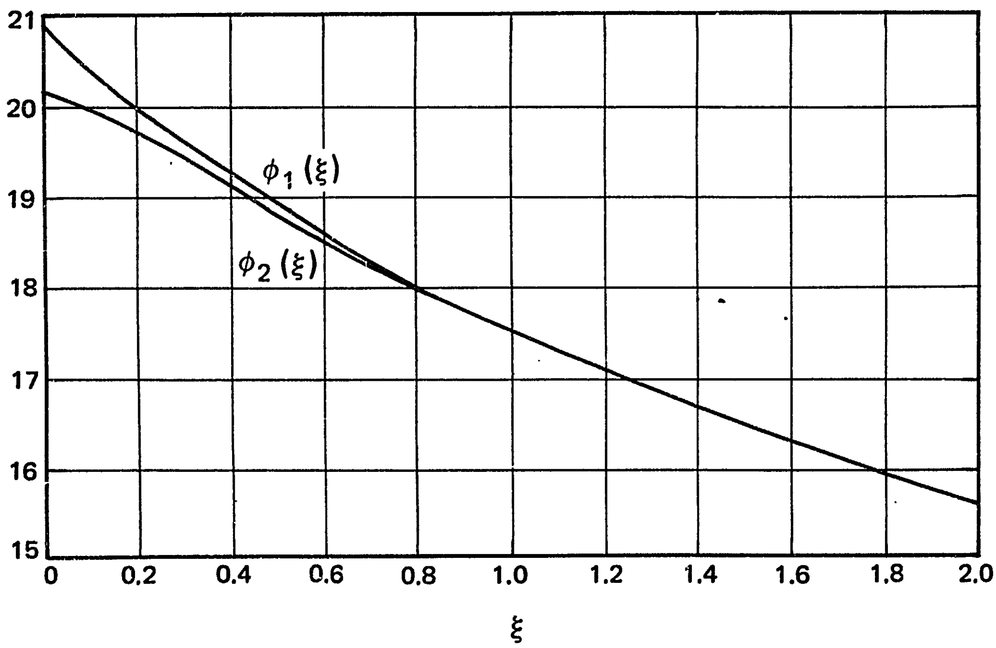

Fig. 2 Screening factors ϕ_1 (ξ) and ϕ_2 (ξ) for the case of intermediate screening II (Eq. (4)) as a function of $\xi = 100k_\nu/E_0EZ^{1/3}$. (from Bethe and Heitler, 1934)

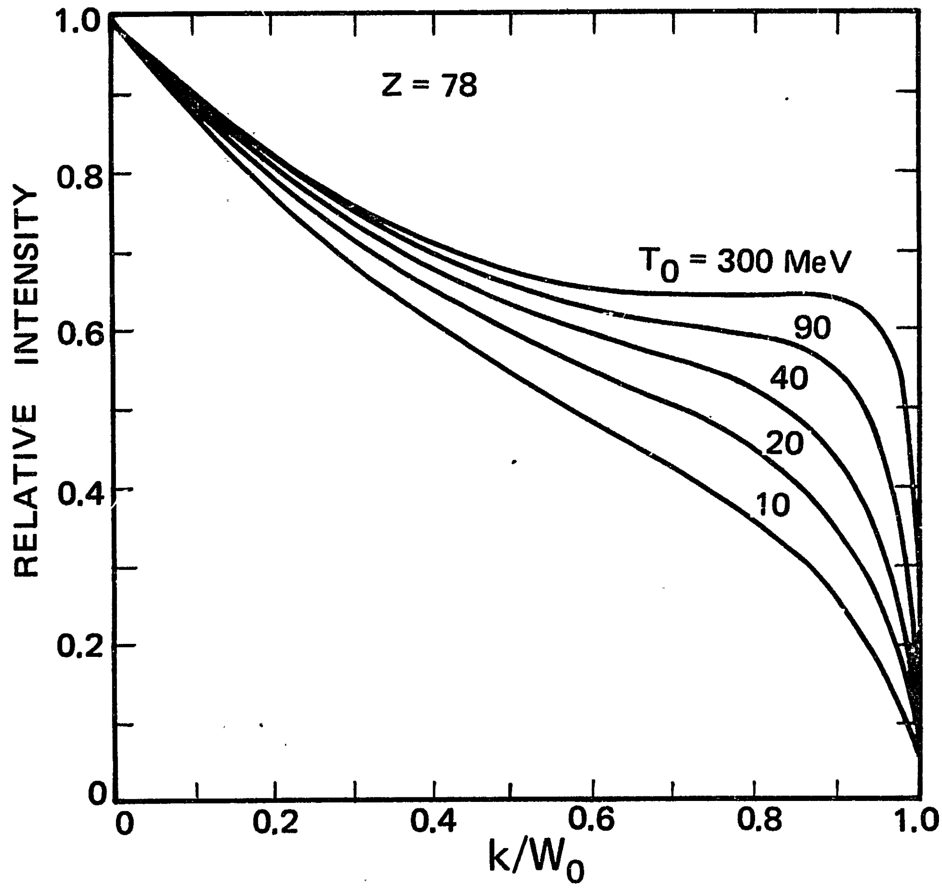

Fig. 3 Dependence of the bremsstrahlung spectrum shape on the electron kinetic energy for a platinum target (Z = 78). W_0 and k indicate electron kinetic energy and photon energy respectively. (from Koch and Motz, 1959)

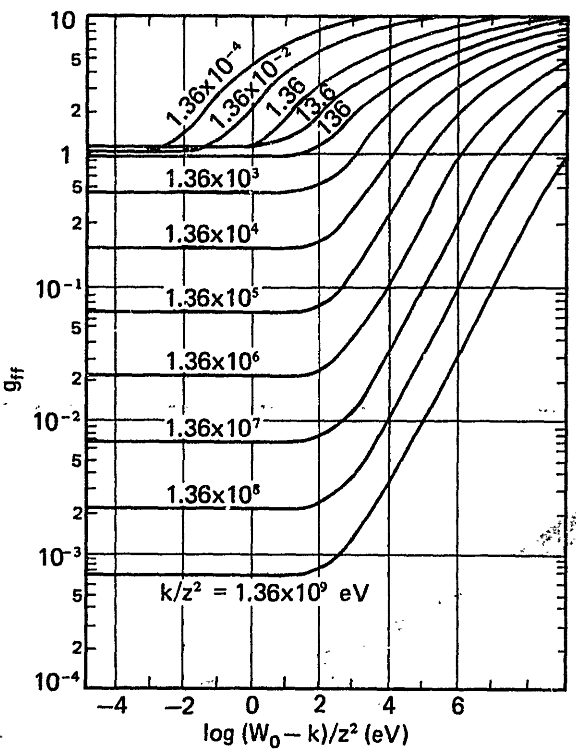

Fig. 4 (a) Free-free Gaunt factor versus final electron energy ($W_0 - k_\nu$) for various photon energies k_ν. (Karzas and Latter, 1961)

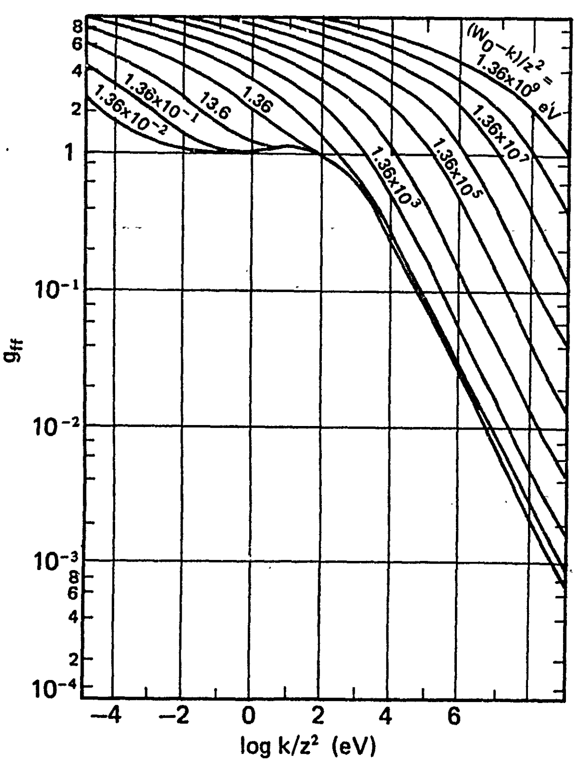

Fig. 4 (b) Free-free Gaunt factor versus photon energy k_ν for various final electron energies $W_0 - k_\nu$. (Karzas and Latter, 1961)

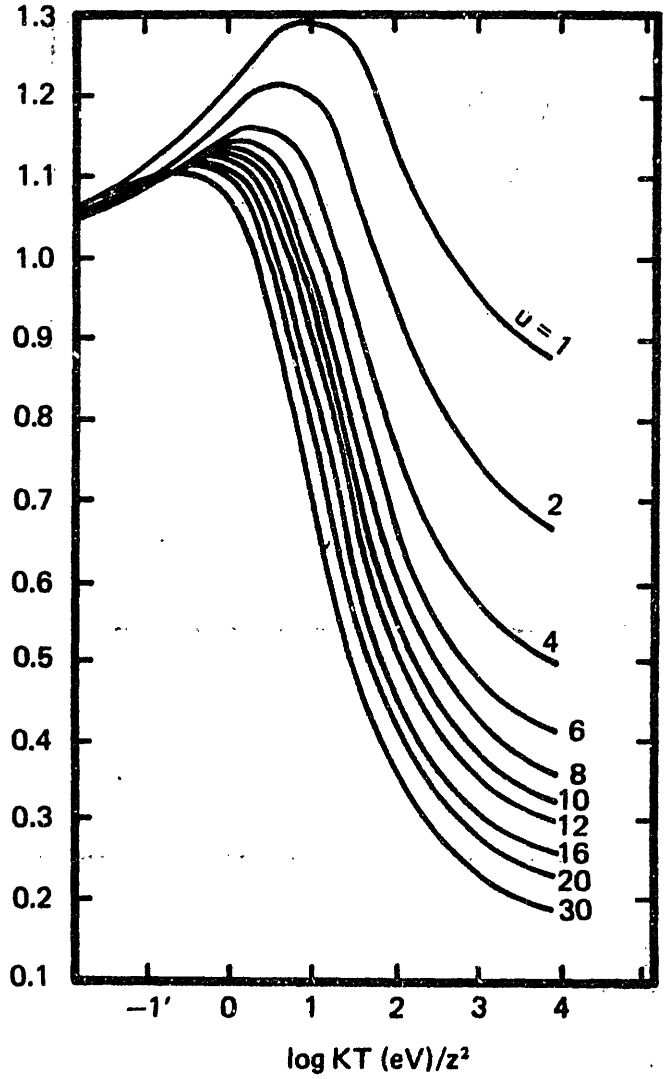

Fig. 5 (a) Temperature averaged free-free Gaunt factor versus kT_e/Z_{eff}^2 for various values of $u = k_\nu/kT_e$. (Karzas and Latter, 1961)

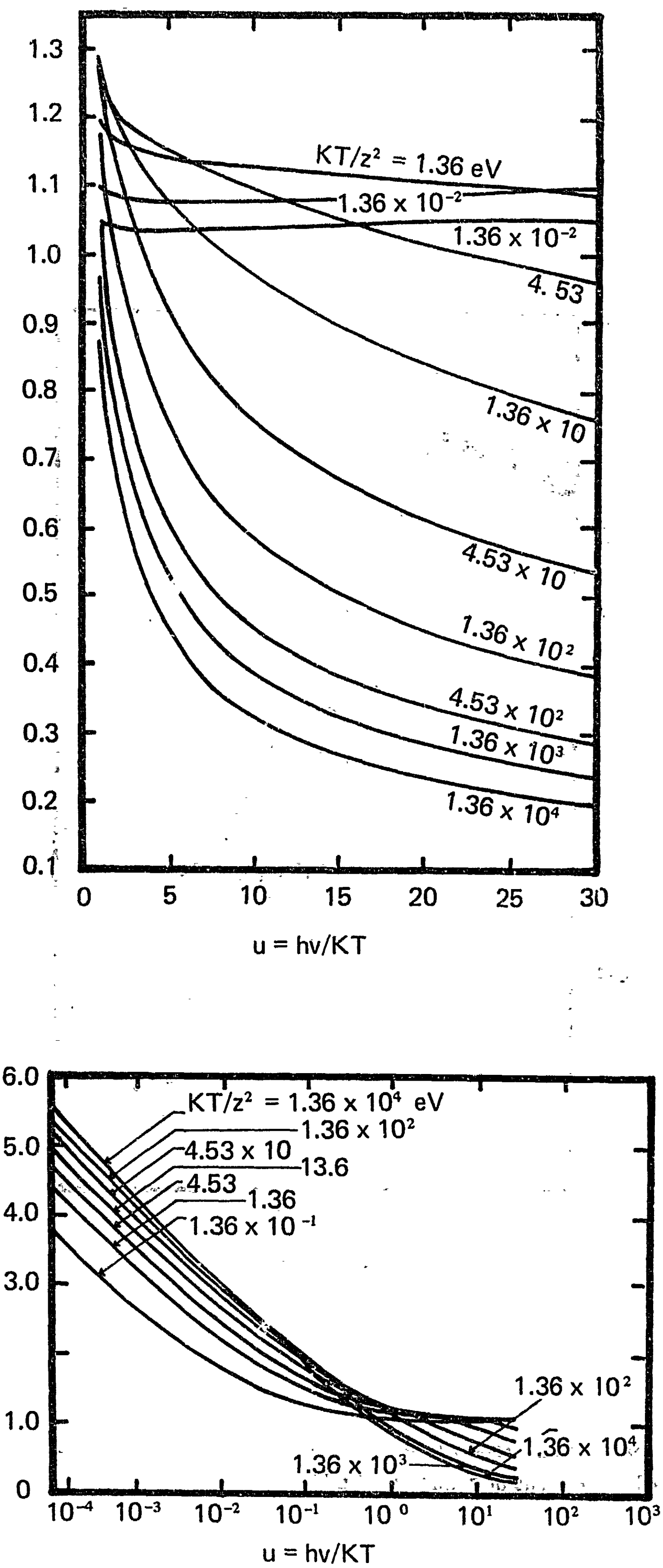

Fig. 5 (b), (c) Temperature averaged free-free Gaunt factor versus $u = k_\nu/kT_e$ for various values of kT_e/Z^2_{eff}. (Karzas and Latter, 1961)

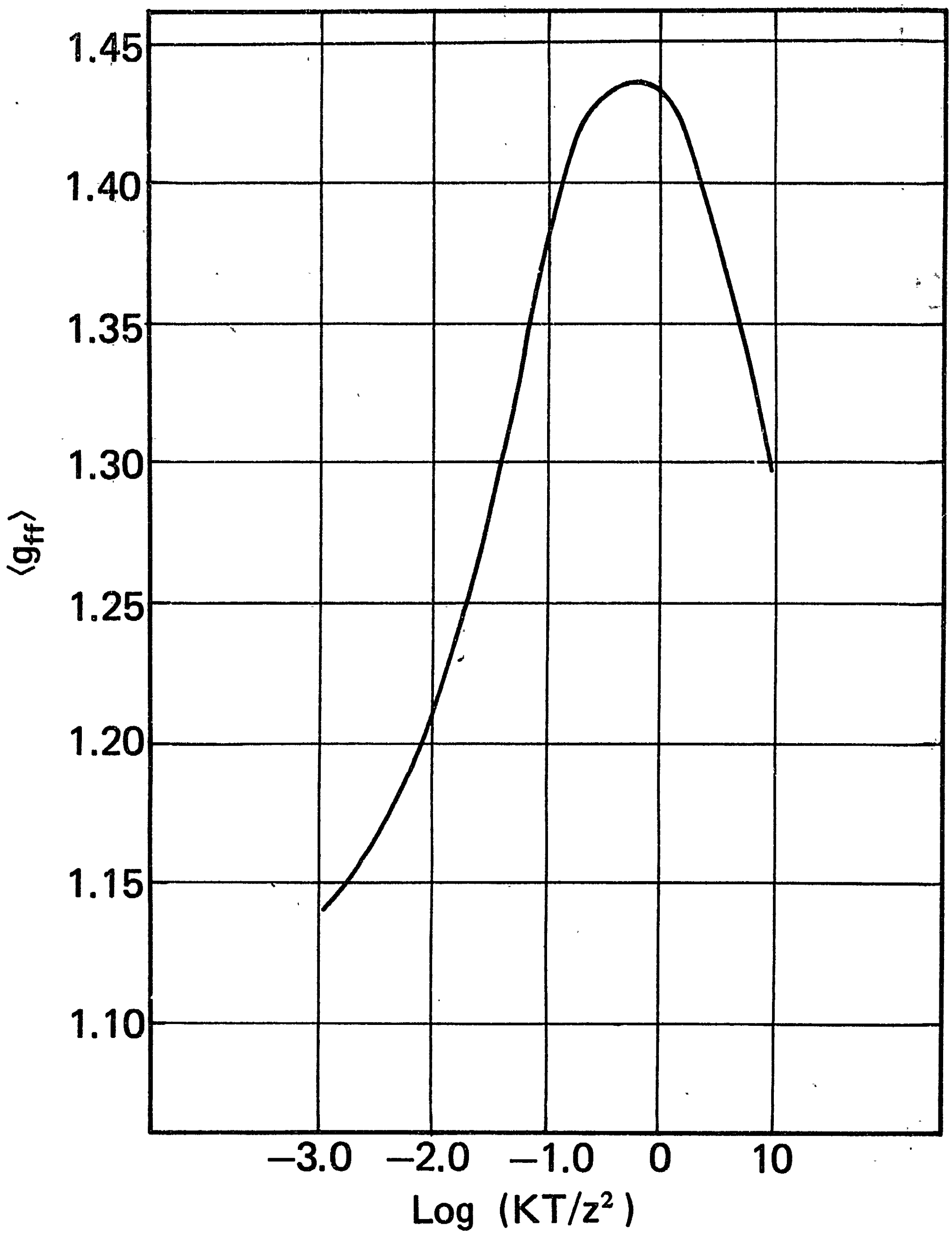

Fig. 6 Integral $\langle g_{ff} \rangle$ of temperature averaged Gaunt factor $\bar{g}_{ff}$ with energy in eg. (17) as a function of temperatures. (Karzas and Latter, 1961)

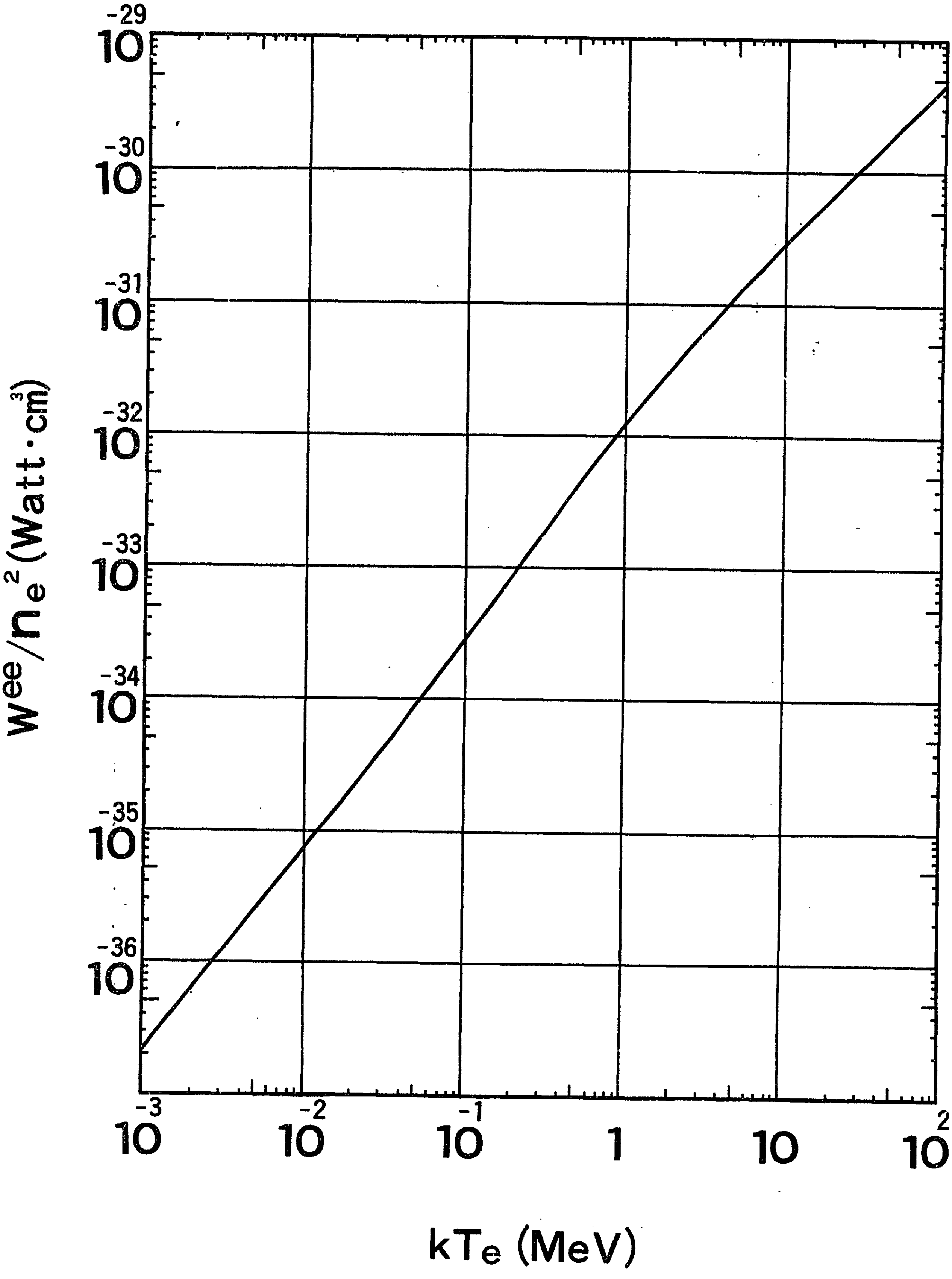

Fig. 7 Total emission rate of electron - electron bremsstrahlung as a function of temperature.

free-free absorption coefficient

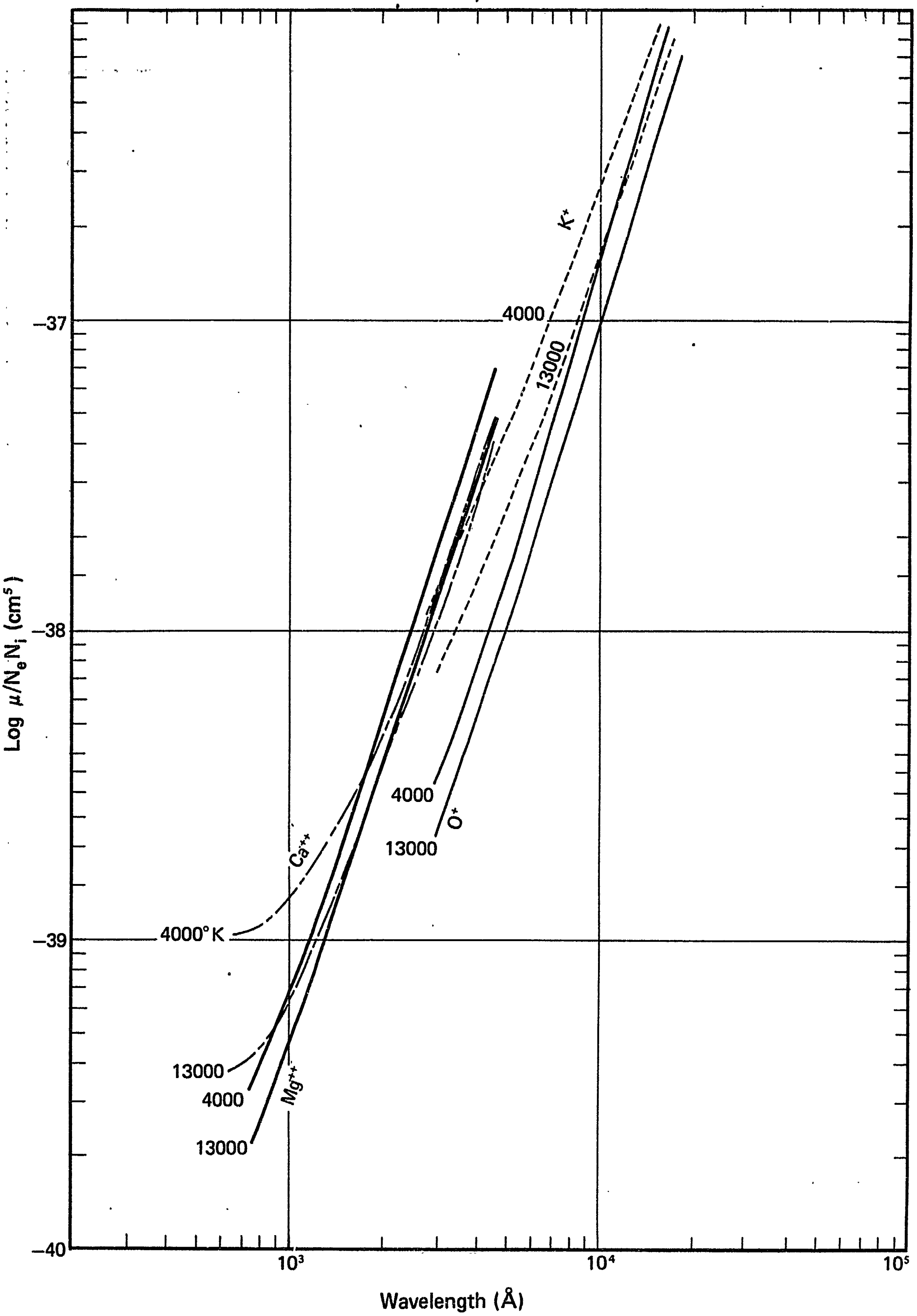

Fig. 8 Free-free absorption coefficients of ions O^+, K^+, Mg^{++} and Ca^{++} for the temperatures 4000°K and 13000°K respectively. (from Peach, 1967)

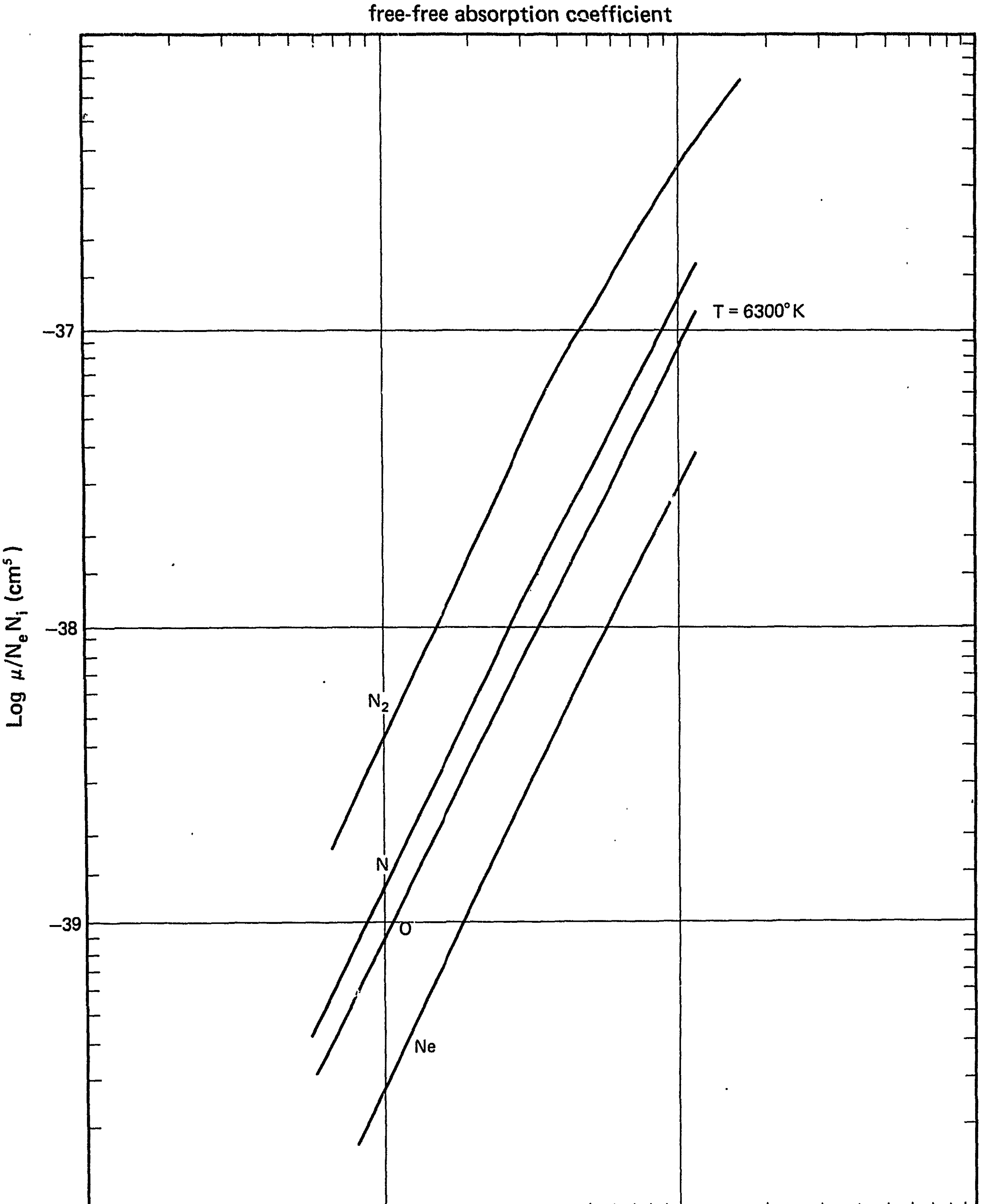

Fig. 9 Free-free absorption coefficients of atoms (N, O, Ne) and of molecule (N_2), for the temperature 6300°K. (from Dalgarno and Lane, 1966)

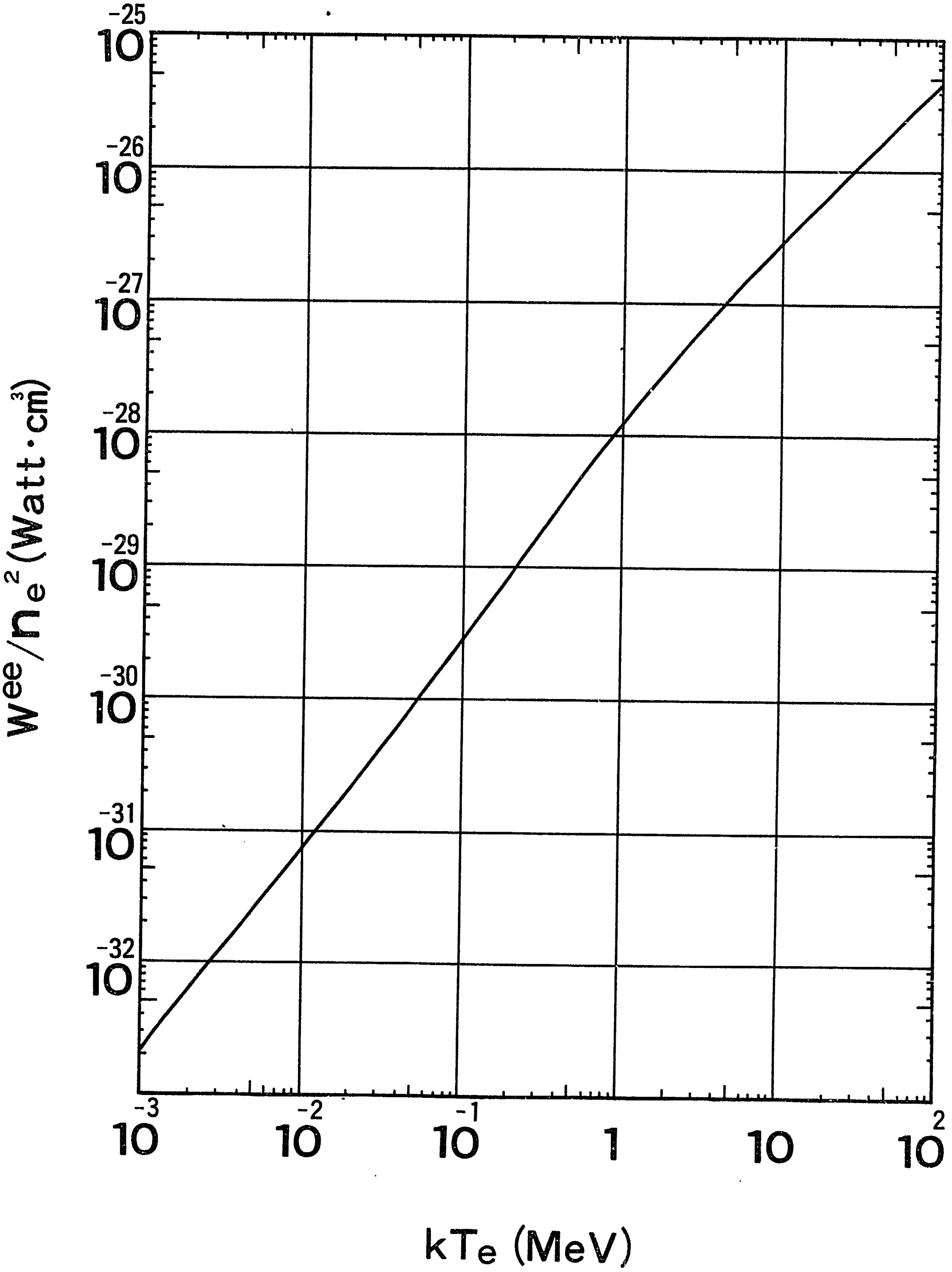

Fig. 7 Total emission rate of electron - electron bremsstrahlung as a function of temperature.

Erratum; The scale of the ordinate of Fig. 7 in IPPJ-AM-6 (1978) is in error by fourth order. Please detach the figure from the enclosed sheet and paste it onto orginal Fig. 7 in IPPJ-AM-6 (1978).